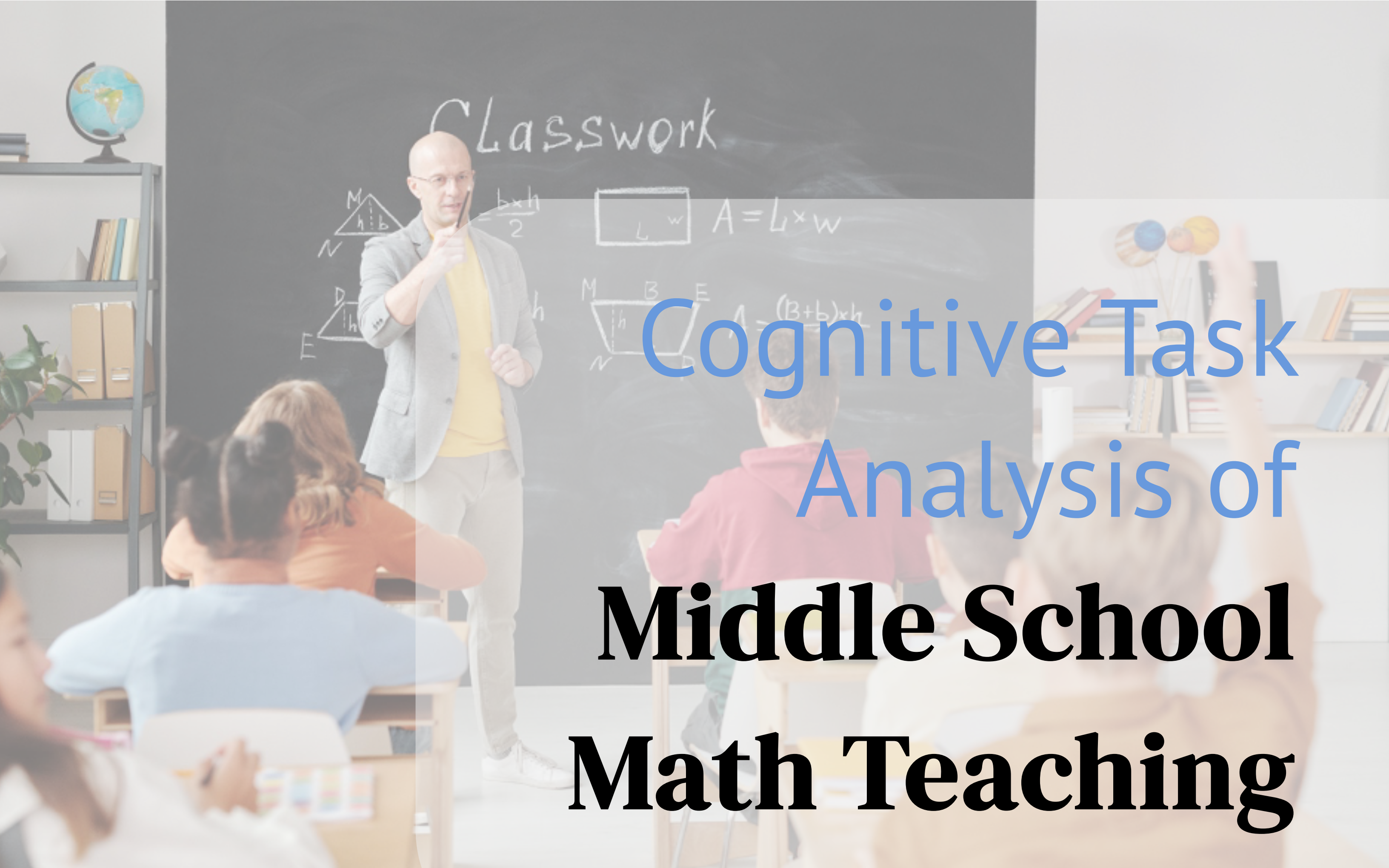

Cognitive Task Analysis of **Middle School Math Teaching**

DRAFT

as of October 5, 2023

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Executive Summary

This report, compiled in partnership with Teaching Lab, addresses the critical need to improve mathematics education in the United States by examining the cognitive work of teachers.

Through application of Cognitive Task Analysis (CTA), we shed light on the cognitive processes that teachers develop and use in middle school math classrooms. CTA is a systematic approach to the naturalistic study of cognitive performance, and this report is the first significant application to middle school math teaching.

We applied CTA with thirty educational professionals with varying levels of experience in teaching middle school math. Half had more than 10 years of experience in education, and several had over 20 years.

The CTA findings reported herein describe the context in which teachers work and the cognitive tasks they attempt to execute in those contexts. We explored the perceived difficulties teachers face and the strategies they use to meet them. And the findings show the trajectory toward acquiring expertise in teaching, and how such experts exhibit the hallmarks of expertise as seen in other domains.

The report concludes with recommendations that focus on creating the conditions for expertise development and technology applications to augment teaching performance. The recommendations are informed not only by the CTA findings, but also by evidence-based guidance for optimizing human cognitive performance from the field of Naturalistic Decision Making.

The significance of this report lies in its potential to help accelerate the achievement of Expert-level performance in teaching and ultimately improve math education outcomes for students.

Acknowledgements

We are indebted to the 30+ educators who shared their professional and personal experiences with us. We hope that we have represented their work, knowledge, and skills accurately. Any mistakes or gaps are ours alone.

We thank Erik Reitinger, Director, Middle Grades Math, and Sarah Johnson, Chief Executive Officer, of Teaching Lab for their partnership and encouragement. We hope that this report marks only the beginning of our relationship.

Finally, we thank the many researchers cited in this report who helped refine our thinking about the cognitive tasks in teaching. We hope that this report serves as a springboard to aid other naturalistic research and development in education and cognitive science.

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Introduction

This report provides the results of a Cognitive Task Analysis of Middle School Math Teaching in the United States.

Context

The findings described in this report were gathered during a moment of tremendous tumult in the profession of teaching. The 2022-2023 academic year was the first full session in the post-Covid era – an era that created significant stress on the educational system that is still being appraised. While the effects of Covid most certainly affected middle-school math teaching, broader trends in student math performance had already been underway. The National Center for Education Statistics (NCES) administered the NAEP long-term trend (LTT) reading and mathematics assessments to 13-year-old students from October to December of the 2022–23 school year. The average scores for 13-year-olds declined 9 points in mathematics compared to the previous assessment administered during the 2019–20 school year. Compared to a decade ago, the average scores declined 14 points in mathematics (NAEP, 2023).

Such outcomes reflect core concerns about the professionals bearing the brunt of systemic strains—strains that are driving a mass exodus. According to a 2022 National Education Association (NEA) survey, "a staggering 55 percent of educators are thinking about leaving the profession earlier than they had planned…regardless of age or years teaching…[and including a] disproportionate percentage of Black (62%) and Hispanic/Latino (59%) educators, already underrepresented in the teaching profession" (NEA, 2022). Workload, income, professional respect, and emergent challenges with student behavior are just some of the oft-cited reasons from those departing.

Still other broadscale trends have added complexity to the landscape. Academic and sometimes politically- and personally-charged public debates have offered guidance and mandates about the 'best' way to teach math, with many stakeholders offering copious evidence in support of their perspectives. Meanwhile, the emerging capabilities offered by large language models of artificial intelligence have rapidly staked claims in the already busy marketplace of educational technologies. Both trends have been marked by hyperbolic claims about the impending or desired 'end' of certain aspects of teaching—and in some cases, the end of teaching altogether.

Goal

Against this backdrop, much research and opining focuses on how students learn, which is tremendously important. A stellar recent example is Koedinger et al. (2023), which found an astonishing regularity in student learning rate and weighty evidence for the idea that, given the right conditions, anyone can learn anything they want.

If we take seriously the importance of "right conditions," then there must also be focus on how teachers teach, how the conditions in which they teach affect their performance, and, ideally, how variations in their approaches affect how students learn.

We have pursued this understanding by implementing a method of study that has proven effective in numerous other domains for unpacking the cognitive performance of skilled professionals—Cognitive Task Analysis.

Approach: Cognitive Task Analysis

CTA is a toolkit used by researchers and practitioners in psychology, instructional design, and system developers to understand how the best performers achieve proficiency. CTA practitioners "get inside the heads" of performers to reveal the nature of their expertise, as it is practiced under pressures including organizational, time, and even safety (Crandall et al., 2006). The CTA toolkit includes techniques for eliciting knowledge, skills and experience; analyzing the revealed data for insights; and representing findings for consumption and targeted application.

Context for a CTA-informed approach

In addition to the demonstrated advantages of CTA in other domains (see *CTA in Effect: Exploring the Return On Investment in Cognitive Task Analysis*), past precedent in educational research strongly suggested the potential value of CTA for understanding middle-school math teaching.

Literature

Cognitive explorations of teaching have been underway since the 1970s. We are indebted to these trailblazers who first saw the need to investigate how teaching actually happens, and we offer here an abbreviated review of the collective wisdom.

Early investigations led by Greeno (1979) applied cognitive task analysis techniques to build an initial understanding of mathematical problem solving in students. Leinhardt and Greeno (1986) later turned attention to the cognitive skill of teaching.

The 1970s also saw the emergence of a decades-long program of research from David Berliner investigating the development of expertise in pedagogy and teaching. Berliner's work was instrumental in describing the characteristics, development and accomplishments of expert teachers—including the use of Dreyfus and Dreyfus' model of expertise development. Berliner's work, principally derived from experiments with teacher participants, provides several analogues for our study. Berliner's 2004 paper, *Describing the Behavior and Documenting the Accomplishments of Expert Teachers*, is a terrific summary of the work and perspective. Other important work has touched on the cognitive work of teaching and mathematics specifically. Rowan (2002) looked at task variety and uncertainty in teaching.

Using CTA to explore teaching and instruction

A growing body of research has implemented CTA as a tool for studying teaching and instruction. Koedinger and colleagues at CMU have conducted a series of studies using varieties of CTA to investigate student thinking (e.g., Koedinger and Terao, 2019).

Richard Clark, Kenneth Yates, and David Feldon from USC in particular have conducted and inspired an extensive program around using CTA to elicit Expert-level procedural thinking in the medical community to include in instructional materials (see, for example, Clark et al., 2008). Their work has been instrumental in demonstrating the large proportion of cognitive performance that is typically *not* included in professional training.

Indeed, the idea of including CTA-derived content in the design of instructional systems has been widely explored. Ryder and Redding (1993) were early adopters, and Bror Saxburg has been a major proponent of the approach more recently. Indeed, Saxburg's work has explicitly called out CTA as a pillar of Learning Engineering, as shown in Figure 1.

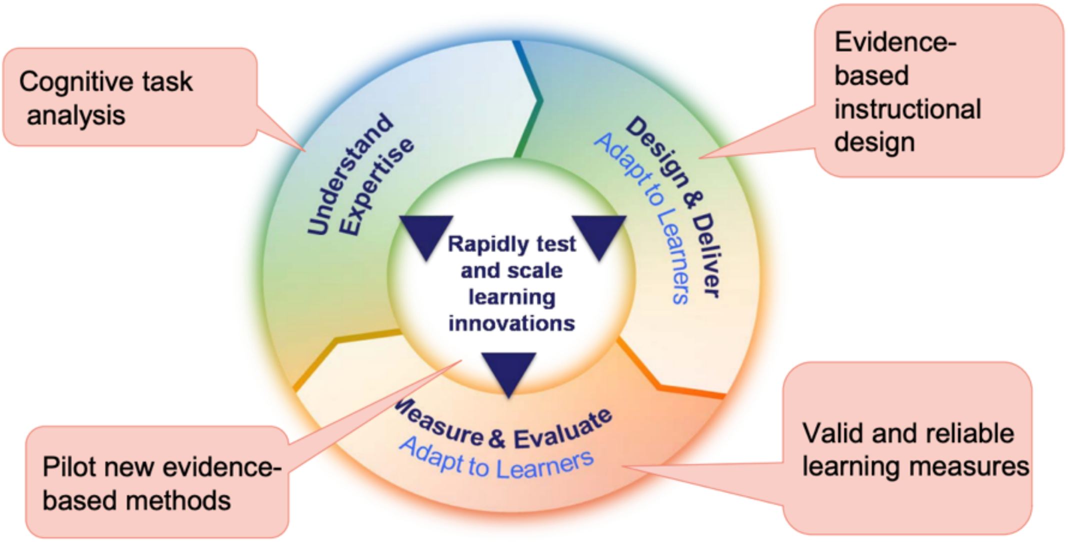

Figure 1: Learning Engineering

More recently, several researchers have called for using CTA to study the practice of teaching more directly. Kartoshkina and Hunter (2014) recommend applying CTA to "add to the existing pool of knowledge about various educational issues by examining cognitive practices and processes of expert educators." Nearly a decade later, Caspari-Sadeghi continued the call, noting that "there exists scarcely any Instructional Design (ID) which is based on empirically deduced knowledge and skills of Expert teachers" (2022).

While scarce, there is an emerging body of work doing just this. Following in the USC tradition, Jury applied "CTA methods to elicit the knowledge and skills expert English teachers use as they teach expository writing to eleventh grade students" (2015). In The Netherlands, van Greel et al. (2018) and Vreman (2019) employed CTA techniques in their investigation of primary and kindergarten teachers' thinking and acting in providing numeracy education. These studies also drew on the USC tradition, which generally aims toward revealing descriptions of procedural knowledge and skills while skimming the surface of "macrocognitive" skills discussed below.

There are, no doubt, other enlightening studies that can inform an understanding of the cognitive work of teaching, and we hope that readers will point us to them. A collective body of such evidence will no doubt prove useful for the future of educational practice.

Perspective

Our study is informed by the Naturalistic Decision Making (NDM) perspective, especially macrocognition (Klein et al., 2003). NDM is a field of study focused on how people *actually* make decisions. The term 'actually' is intended, as NDM researchers use CTA methods to study work as it is performed in context—not in the limited confines of a laboratory. Using this approach, NDM researchers have discovered and specified a number of models of cognitive performance collectively known as macrocognition. While the number and inclusion of such models has evolved over the past two decades through new research, the basic paradigm has remained unchanged. By studying performance in its natural occurrence, NDM researchers have thrown light on the nature of expertise. Where CTA provides the methods for NDM, macrocognition provides the conjectural models for exploring any domain. Our study drew inspiration from macrocognition through data collection and analysis.

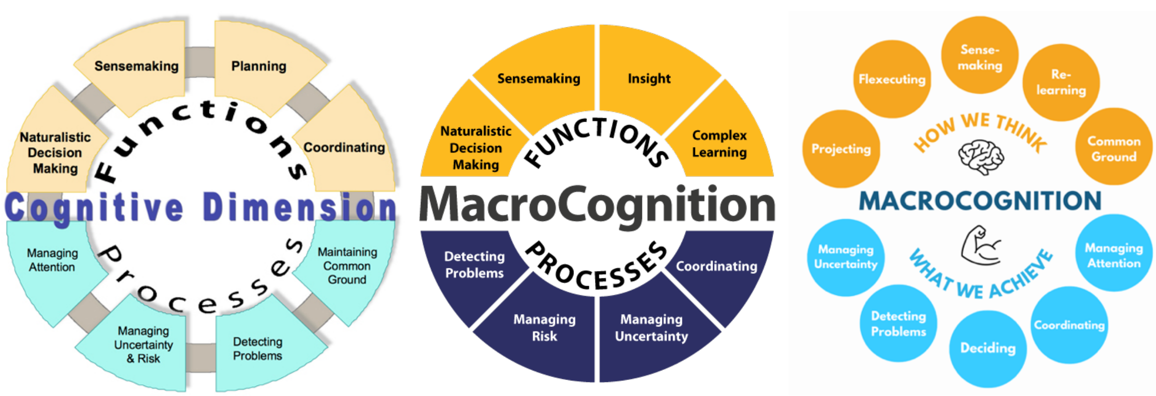

Figure 2: Macrocognition

NDM researchers have revealed how expertise develops, which is fundamental to helping people achieve it. Mastery Models of skilled performance build on the Dreyfus and Dreyfus model of expertise development to generate a representation of development in any domain characterized by complex expertise (Ross and Phillips, 2019). They include customized stage profiles for key areas specifying the hallmarks of performance and characterizing the progression of skill, and performance indicators for each area in five progressive levels of development. Not only were we inspired by the structure of Ross and Phillips' Mastery Models, but it so happens that their exemplar model was that of Marine Corps Instructors, thus providing a relevant precursor to our data analysis and representation approach.

Challenges for Studying the Cognition of Teaching

Conducting a CTA of middle school math teaching presented several challenges.

Defining the "task" of teaching

Cognitive Task Analysis has, as implied, traditionally focused on the performance of cognitive *tasks*. Most applications have focused on tasks within a domain, such as recognizing sepsis in infants and detecting landmines in military force protection. Teaching is not 'a' task. It is a role, a profession, comprising a set of temporally interwoven tasks, each with its own set of complexities. Studying "teaching" is akin to studying "doctor-ing." While it is possible to gain a deep understanding of the full range of tasks in a domain given extensive access to practitioners (see for example, *Minding the Weather*), our study accessed only a portion of the many tasks inherent in middle school math teaching. This said, the CTA toolkit enabled the discovery of a set of cognitive tasks that comprise the core of teaching performance. We are as confident that others could be discovered as we are that we have hit upon a basic set.

Methodological limitations and revised approach

The CTA toolkit offers a set of methods for helping professionals articulate their experience. It includes methods for examining specific incidents, i.e., Critical Decision Method (CDM), and also for describing the component experiences, knowledge, and skills within tasks, i.e., Knowledge Audit. CDM is particularly useful for revealing cognitive performance in the context of challenging incidents. CDM can be particularly useful with experienced practitioners who can draw on a large bank of such incidents – it is less useful with junior practitioners. We regularly use CDM to explore cognitive tasks where there is some uniformity in the participant pool with respect to task performance so that the collective of incidents is comparable. In this case, our participant pool was quite diverse, with participants currently working in teaching, coaching, administrative, and tutoring capacities.

In the practice of teaching, the concept of an "incident" can be difficult to define. While we were able to successfully use the CDM to elicit stories about working with *individual* students and a whole-school turn-around story that was one of the most enlightening we captured, the continuous flow of teaching experience and the large numbers of "cases" (i.e., students, teaching units, and for more experienced teachers, years of practice), suggested that an incident-based approach would limit participants' ability to articulate their cognitive performance. We have seen a similar situation during a large-scale CTA with healthcare professionals – doctors, nurses, and administrators (Moon & Hoffman, 2014). We adjusted our CTA approach similarly here, more toward "cognitive interviewing" using Knowledge Audit probes and capturing exemplars as they helped illuminate aspects of cognitive performance. A general set of tasks, tensions, and macrocognitive processes emerged, enabling us to verify but revise these findings with each subsequent interview.

Importantly, we conducted no observations. Nor did we have direct access to data regarding teacher performance or student outcomes for most of the performance cases we reviewed. We suspect observations of teaching practice and access to data would provide further illumination on our findings.

Defining "expert teaching"

There is no shortage of perspectives on what makes for "expert teaching". Mccrea's resources (2023) offer an insightful, process-based perspective on some of the components of expert teaching. Leinhardt and Greeno (1986) defined their version of expert teachers by the *outcomes* of their teaching—i.e., growth in student scoring data measured over time. Berliner (2004) took a professional credentialing approach, arguing that the best way to identify experts is to let the profession ferret them out.

Each of these approaches for determining expertise is a reasonable way to determine who could be considered an expert teacher. In the NDM community, Hoffman (2023) has suggested The Pentapod Principle for determining proficiency in any domain:

Always use at least two and ideally three methods
from the distinct methods classes
to converge upon and validate a proficiency scale
that is appropriate to the given domain.

Hoffman suggests five such method classes, and below we note how we determined which were appropriate for our study:

Table 1: Proficiency Determination Methods

In-depth career interviews	We did not conduct these for all, but did collect and inquire about years on the job and roles held. We also captured the context in which they gained experience, especially noting when such context was marked by extreme organizational pressures, as such context is known to be key for developing expertise.
Professional achievements, standards, or licensing	We identified some of these for some participants, but none of our participants were nationally certified.
Measures of performance at the familiar tasks	Where access was available, we collected student outcome data; however, in only a few cases were the data directly attributable to performance.
Social Interaction Analysis (Sociometry)	This approach was not available to us.
Cognitive Task Analysis	We used this method extensively.

Regarding the use of CTA, we developed an initial Mastery Model of expertise development after ~15 interviews, then used it as a point of comparison for all participants *in order to assign them a proficiency level*. This approach helped us to both revise the Mastery Model as new aspects became apparent and to consider whether participants exhibited attributes and performance at each level so that we could determine their level accordingly. That is, once we identified the characteristics and associated performance indicators of the various levels, we sought to identify whether such indicators were offered (with or without prompting) during the CTA interviews. If higher-level indicators were not offered, we determined that participants could not be placed at higher levels of proficiency. The rationale for doing so is straightforward: if a performer does not even think of the difficulties, or appreciate the complexities, or consider the information sources, or think to execute – *in ways that higher-level performers do* – then that performer cannot be considered for higher proficiency.

Of course, this approach can only be implemented once a general-but-tentative structure for the Mastery Model is developed. We already have a good sense of what is to be expected at the various levels of proficiency based on known characteristics and attributes from other domains. As reported by Ross and Phillips (2019), the levels are:

Table 2: Generic Proficiency Levels

Novice	• Lack of experience with real-world situations • Little situational perception • Rule-based or procedure-based performance • Abstract thinking without contextual anchors • No discretionary judgment
Advanced	• Some experience with real-world situations, enabling recognition of recurring elements • Internalized guidelines for action based on response to limited attributes or aspects they have learned to recognize • Situational perception limited • Situational attributes and aspects are treated separately and given equal importance • Pattern recognition absent
Competent	• Sees action at least partially in terms of longer-term goals • Conscious, deliberate planning • Skilled at formulating goals and plans • Manages large amounts of information well • Standardized and routinized procedures • Plan guides performance as situation evolves and fails to adjust
Proficient	• Sees situation holistically rather than in terms of aspects • Pattern recognition • Sees what is most important in a situation • Perceives deviations from the normal pattern (anomalies) • Automatic and dynamic situational assessment based on experience • Situational factors guide performance as situation evolves • Requires analytic deliberation to reach course of action decision
Expert	• Rules or maxims entirely internalized • Focuses on only critical elements • Intuitive situational assessment based on deep tacit understanding • Intuitive recognition of appropriate decision or action • Analytic approaches used only in novel situations or when problems occur

Even more can be said of the Expert-level, as Klein and Hoffman noted decades ago (1992). Expert performers have the ability to see typicality and make fine discriminations. They 'see' antecedents and consequences. We also know that experts characterize problems in a fundamentally different way from those with less experience (Moon et al., 2010), and that achieving expertise requires undergoing a metamorphic process that involves execution and feedback, recognition and analysis of one's own performance compared to others, and submission to people and processes identified with Expert-level performance, if they are available (Moon and Bildstein, 2019). Importantly, we saw all of these characteristics in our designated Expert-level teachers.

Berliner (2004) used similar levels to set up an analogous stage model for teaching, which we found very useful. At the Expert-level, Berliner proposed we should expect to see:

- better use of knowledge;
- extensive pedagogical content knowledge, including deep representations of subject matter knowledge;
- better problem-solving strategies;
- better adaptation and modification of goals for diverse learners and better skills for improvisation;
- better decision making;
- more challenging objectives;
- better classroom climate;
- better perception of classroom events and better ability to read the cues from students;
- greater sensitivity to context;
- better monitoring of learning and providing feedback to students;
- more frequent testing of hypotheses;
- greater respect for students; and
- display of more passion for teaching.

This foundational set of indicators served as a signpost for describing the Expert-level of performance in our Mastery Model, and we have added detail to and extended this set with our cognitive task analysis.

Finding experts

It's one thing to know what experts look like – finding them is another. Working with Teaching Lab, we conducted a snowball sampling approach to registering candidate participants. We started with our networks and sought participants with varying levels of time on the job, with a goal of including a significant proportion of the participants having significant experience in teaching middle school math.

We also intended to reverse engineer the "measures of performance at the familiar tasks" approach by using The Educational Opportunity Project at Stanford University to identify high-performing schools within otherwise low performing districts and contacting administration officials to put us in touch with their identified 'best' teachers. Our response rate with this approach to this point has been low.

Thus, as of this draft report, our participant pool comprises the following:

Table 3: Participant Experience Levels

	Years of experience teaching and/or administration	Years of experience teaching MS math
20+	8	4
11-19	6	6
6-10	12	10
0-5	3	9
Only tutoring	1	1

While we have not yet achieved the goal of skewing toward high levels of experience, one-third of our participants had significant experience teaching middle school math and nearly one-half had significant teaching experience.

Notably, we characterized as expert only one of our four participants with over 20 years' experience teaching middle school math. Across the levels, our estimations of proficiency did not track exactly with the years of experience.

Table 4: Participant Proficiency Levels

Expert	4
Proficient	14
Competent	10
Advanced Beginner	2
Novice	0

While the four participants we designated as experts in teaching MS math had 20+ years of experience *in education*, only one had more than 20 years of experience *teaching middle school math* (27 years). The other three averaged ~13 years of experience teaching middle school math, suggesting that the process of obtaining Expert-level performance *can be accelerated***.** That is to say, *under the right conditions*, it is feasible to reach Expert-level performance in just over a decade.

With the remaining CTA interviews, we hope to capture more Expert-level findings. Fuller descriptions of our participants are provided in the next section.

Value and Importance

Our report is the first attempt we know of to conduct a large-scale CTA of any teaching practice. With these findings, we are building on the evidence base of findings about the cognitive practice of teaching middle-school math. CTA—informed by Naturalistic Decision Making—enables us to sharpen details around prior, related reporting, and offers a fresh perspective, including specifying a Mastery Model that refines and updates Berliner's expertise model (2004).

Our findings do not, however, offer endorsements about pedagogical techniques or philosophies. We have not explicitly linked our findings to outcome data in a broadscale way, though one incident from an Expert-level performer offered a very strong case in support of the relationship between expert performance and student outcomes. This in our analysis gap precludes endorsement, so it is possible that proponents of particular approaches will find both support for and critiques of their positions in our evidence base.

Despite the noted limitations regarding the number of expert teachers we have worked with thus far, our exploration of the Mastery Model levels suggests that it is possible to demonstrate differences in the way performers at the various levels go about their work. This is the necessary first step to defining requirements for support at each level and strategies for professional development for bringing performers to higher levels more rapidly than would otherwise happen.

Given the other-than-math-teaching experience of many of our participants, we can suggest with confidence that many of our findings will generally hold outside of the context of middle school math. We hope that others will pursue similar investigations in other topics so that this hypothesis can be assessed.

Our primary finding lines up with prior investigations: Teaching is difficult. Developing into an expert teacher is extremely difficult and rare. Creating the conditions for expertise development in teaching should be a key goal of educational institutions.

Report of Findings

Participants

Given the method of selection described above, our participants are not a representative sample of middle school math teachers in the United States. That said, they represent a diverse sampling with respect to the key demographic for this study, experience in teaching middle school math.

Table 5 provides a summary of our participants' experience.

Table 5: Summary of Participant Demographics

Employer State	10, with significant representation from NY	
Primary School Type	Charter	15
	Public	12
	Other	3
Primary Role at time of interview	Teacher	16
	Administrator	9
	Other	5
Education	University	4
	Post-Graduate (including in-process)	19
	Other or N/A	7

Table 6 provides details about our participants.

Table 6: Detailed Participant Demographics

State	Roles	Academics, Training and Certifications	Years EDU	Years MSM	Level
NY	Classroom teacher (Grade 5) at CHARTER, Classroom teacher (Grade 9 Biology) at CHARTER in NJ, Center Director at after-school enrichment provider), Classroom teacher in MD PUBLIC	Currently working towards a Master's degree in Teaching	10	1	C
NY	Classroom teacher at CHARTER (Grade 5)	Graduated from St. Francis College with Bachelor's in Education	10	5	C
NY	Classroom teacher (Grade 8), Senior Manager of Academic Data Analytics at CHARTER	Earned a Postgraduate Certificate in Education from the University of Bristol	9	5	P
NY	Classroom teacher (Grade 6), Assistant Principal for Math & Science at CHARTER	Teach for America; Graduated from Relay Graduate School of Education with a Master's degree in Teaching; Earned a Postgraduate Certificate in Instructional Coaching from the University of Pennsylvania Graduate School of Education	9	9	C
NY	Director for Middle School Math at CHARTER, Classroom teacher (Grade 7) at CHARTER, Classroom teacher (Grade 5) at CHARTER, Dean of Instruction at CHARTER, Classroom teacher (Grade 7) at CHARTER	Graduated from Relay Graduate School of Education with a Master's degree in Secondary Education and Teaching	9	9	P
NY	Dean of Curriculum and Instruction, Curriculum writer, Instructional leader at CHARTER	Graduated from the University of Albany with a bachelor's degree in Education; Graduated from Relay Graduate School of Education with a Master's degree in Teaching	5	4	C
NY	Director of Math for Grades 5-8 at CHARTER, (Grade 7) at CHARTER, Dean of Curriculum & Instruction at CHARTER	Graduated from MIT with a Bachelor's degree in Management Science	14	5	P
VA	Math curriculum writer for VENDOR, Classroom teacher (Algebra 1 & Geometry) for PUBLIC, Classroom teacher (Grades 5-7) at PUBLIC l, Substitute teacher at PUBLIC, Math Tutoring Research Assistant at UNIVERSITY	Graduated from the University of Virginia with a Bachelor's degree in Mathematics; Graduated from Liberty University with a Master's degree in Secondary Mathematics & Teaching	8	7	P
VA	Interim Assistant Principal at PUBLIC, Classroom teacher at PUBLIC, Classroom teacher (Grade 6, 7, 8) at PUBLIC, Classroom teacher (Grades K-2) at PUBLIC, Math department chair at PUBLIC, SOL item and test review committee member with STATE, Mathematics and curriculum framework revision committee member with STATE	Graduated from the University of Pittsburgh with a bachelor's degree in Elementary Education; Graduated from Virginia Commonwealth University with a Master's degree in Administration and Supervision; Statewide communities of Practice for Excellence Scope 10 Graduate from the University of Virginia	16	13	P
CT	Chief Academic Officer at PUBLIC, Classroom teacher (Grades 5 & 6) at PUBLIC	Graduated from Western Connecticut State University with a PhD in Instructional Leadership	32	5	P
NM	Member of Dual Language Education, Classroom teacher (Grades 6-9) at PUBLIC	Graduated from College of Santa Fe with a Master's degree in Curriculum & Instructional Leadership	20	16	E
NY	Senior Director of K-12 Math at CHARTER, Curriculum and Instructional Consultant at CHARTER, Math department chair and Classroom teacher (AP Calc) at CHARTER	Earned a High School Math Teaching Certificate from Relay Graduate School of Education	16	6	C
CA	Math specialist at CHARTER, Classroom teacher (Grade 8) at PUBLIC	Graduated with a Master's degree in Education; Earned a Teaching Credential with an emphasis in Mathematics and Multicultural Learners	22	12	E

NY	Professional Learning Specialist at VENDOR, Professional Development Provider at CHARTER, Grade level chair at CHARTER, Classroom teacher (Algebra 1) at CHARTER, Teaching Policy fellow at CHARTER, Classroom teacher (Grade 8) at PUBLIC, Middle school lead and Mathematics Department chair at PUBLIC	Teach for America; Graduated from Marian University Indianapolis with a Master's degree in Secondary Education and Teaching	6	6	C
NY	Classroom teacher (Grades 7-9) at PUBLIC	Graduated from the State University of NY at Fredonia with a Bachelor's degree in Elementary Education and Teaching; Graduated from Buffalo State University with a Master's degree in Elementary Education and Teaching	18	18	P
NY	Special Education classroom teacher (Grades 6-7) at PUBLIC, Teaching assistant at PUBLIC, Special Education Intern at PUBLIC, Teacher assistant intern at PUBLIC	Graduated from SUNY Geneseo with a Bachelor's degree in Education/Teaching of Individuals in Early Childhood Special Education Programs	1	1	AB
SC	Site Director for VENDOR, Classroom teacher (Grades 8-9) at CHARTER, Substitute teacher at CHARTER	Graduated from Stockton University with a Bachelor's degree in Psychology and Mathematics	7	1	C
MD	Instructional Specialist in Secondary Mathematics at PUBLIC, Manager in Secondary Mathematics at District of PUBLIC, Specialist in High School Mathematics at PUBLIC, Director at UNIVERSITY, Secondary Mathematics PDS Coordinator at UNIVERSITY, Classroom teacher (Grades 6-9) at PUBLIC	Graduated from Duke University with a Bachelor's degree in Mathematics; Graduated from Boston College with a Master's degree in Education - Mathematics; Graduated from the University of Maryland with a Master's degree in Education Policy and Leadership; Graduated from the University of Maryland with a doctoral degree in Education Policy and Leadership	23	10	E
NJ	Classroom teacher (Grade 7) at CHARTER	Graduated from Bloomfield College with a Bachelor' degree in Mathematics	2	2	AB
TN	Classroom teacher (Grade 5) at CHARTER	Graduated from Lipscomb University with a Bachelor's degree in Elementary Education and Teaching	19	17	P
VA	Math specialist (Grades K-8) at PUBLIC, Classroom teacher (Grades 5-8) at PUBLIC	Graduated with a Master's degree in Education	25	22	P
PA	Special Education classroom teacher (Grades 6-8) at PUBLIC	Graduated from St. John's University with a Master's degree in Teaching Children with Disabilities in Adolescent Education (7-12)	8	8	P
PA	VENDOR in tutoring for 25 years	Graduated from University of Pennsylvania with a Master's degree in Sociology	0	0	C
MD	Content specialist at PUBLIC, Classroom teacher (Grades 6-8) at PUBLIC	Graduated from the University of Maryland with a Master's degree in Teacher Leadership	23	23	P
NY	Classroom teacher (Grades 7-9) at PUBLIC	Graduated from SUNY Brockport with a Master's degree in Secondary Mathematics	20	20	P
NY	Teacher apprentice (Grades 5-8) at CHARTER	Graduated from the University of New Hampshire with a Bachelor's degree in Mathematics	6	6	C
SC	Tutor at VENDOR, Classroom teacher (Grades 5-9) at PUBLIC	Graduated from University with Bachelor's degree in Education	6	6	C
NY	Classroom teacher (Grade 8) at CHARTER, Classroom teacher (Grade 6) at CHARTER, Classroom teacher and grade level lead at CHARTER, Interim Principal at PUBLIC, Classroom teacher (Grades 6-8) at PUBLIC	Graduated from Columbia University with a Master's degree in Education Policy Analysis	11	11	P
MD	Instructional Specialist at PUBLIC, Classroom teacher (Grades 6-8) at PUBLIC, Math Content Specialist at PUBLIC, Grade-level Team Lead at PUBLIC	Graduated from Towson University with a bachelor's degree in Middle School Education - Mathematics and Science; Graduated from Hood College Graduate School with a Master's in Educational Leadership	9	7	P
VA	Mathematics Department Chair at PRIVATE, Classroom teacher (Grades 5-8) at PRIVATE	Graduated from Smith College with a bachelor's degree in Mathematics; Graduated from Johns Hopkins Univ with a Master's degree in Numerical Science	27	27	E

Multiple Views of Cognitive Performance in Middle School Math Teaching

The CTA findings provide several complimentary views of the cognitive work of teaching middle school math, including:

- The context in which teachers conduct the work
- The cognitive tasks they attempt to execute to do the work
- The mastery of the work they achieve over time.
- The macrocognitive processes that experienced teachers use to achieve the work.
- The difficulties encountered, and strategies used to achieve work, in subdomains.

The following sections present these views.

The context in which teachers conduct the work

From an NDM perspective, the context of work refers to the pressures and tensions that performers face. Whether they are explicitly stated or implied, the context of the work imposes requirements that confront workers, introduce complexity, and shape their strategies.

The CTA findings revealed 11 aspects and three emergent features of context within which middle school math is taught. We discovered these aspects across participants, and thus across situational aspects – i.e., public/private schools, urban/rural settings. They continuously challenge teachers to direct their attention, place emphasis, design strategies, and assess their own performance and development. To be clear, these aspects of context are not imbued with any value judgment. Reflecting them serves only to throw light on the broader context in which cognitive work happens. Understanding these contexts is critical to appreciating the cognitive demands of teachers.

Goal: Understanding | Performance

Middle school math teachers perpetually consider what *does* and what *should* define student achievement. This tension is between understanding and performance, between ensuring that students gain an appreciation for the complexity of mathematics and ensuring that they can meet the moment when called to action. This aspect shapes decisions about where to focus and suggests where emphasis should be placed and in which moments.

Endgame: Conceptual | Procedural

Hand-in-hand with the goal is the consideration of the ultimate requirement for learning math. This tension is between the conceptual and the procedural. While this tension is not unique to math, it is ever present in the policy discussions and practical concerns that surround math instruction—see for example, the math war that has been raging for decades. The aspect shapes planning for what to *try* and how to *try* it.

Curricula: Provided | Created

Curricula instantiate the goals of instruction and advise on what counts as achievement. This tension is between what is provided and what is created to meet the goals and achieve them. It is ratcheted up by decisions made outside of one's control (e.g., vendor selection), frequent revision and overhauls, and mandates imposed across several levels of oversight (e.g., in sequencing). This aspect shapes planning for what to *do* and how to *do* it.

Pacing: Planned | Executed

Teaching is generally governed by seasonal ebbs and flows but managed by pacing schedules. This tension is between the plan as stated and the plan as executed, the delta of which may be tracked with more or less precision. The aspect shapes planning and decision making for when to try and do, and for how long.

Coverage: Comprehensive | Prioritized

Various standards determine the content to be covered in the course of any given school year. This tension is between ensuring that all content is covered and prioritizing which content should take priority in the context of time pressure. The aspect shapes planning and decision making for how long to try and do.

Target: All | Individual

Middle school math teachers must continuously consider their target of focus. The tension is between all—which can mean the entire class or subsets—and the individual student. Attention toward one or the other runs the risk of denying attention to the other. This aspect shapes the decision making and planning process for determining what to try and do, when, and for how long.

Tools: New | Old

While differences in resource levels exist, all middle school math classrooms are infused with tools. This tension is about the value introduced by new and old technologies, where value is considered according to judgments about potential efficiencies, perceived learning gains or other student-centered benefits, and the degree to which use is mandated. The aspect shapes planning and decision making for what to try and do, when, and for how long.

Evaluation: Process | Outcome

Teachers have only indirect access to highly variable data with which to monitor and evaluate their own performance, including data about the performance of their students. This tension is between whether process or outcome data should be used to craft the stories that characterize their own performance. Each may be more or less appropriate for different purposes and audiences. This aspect shapes the sensemaking process for determining how the work is going.

Data: Volume | Insight

The outputs of evaluations are data about student performance, and at any given moment, the teaching process faces a *potentially* overwhelming deluge of data. This tension is between getting more data or getting insight. More data can overwhelm professionals, especially if they are not trained in how to interrogate it, and supposed insights can often be countered with the right data. This aspect shapes the planning process for determining what to do about how the work is going.

Responsibility: Students | System

Teachers mediate the relationship between students and the system – i.e., policies and the administrators that devise and enforce them. This tension is between the needs of students and the demands of the educational system. While every student is different and under continuous evolution, the system can be just as fickle—and both can misalign with frequency. The aspect shapes decisions about which entity receives emphasis at any given moment.

Autonomy: Self-determined | Constrained

At any given point in their careers, teachers are endowed with different levels of autonomy. This tension is between determining one's courses of action and constraints imposed by others. Both circumstances offer advantages for teachers and students, yet both can introduce challenges, which are felt most urgently when circumstances flip. This aspect shapes decisions and planning about what to do, when, and for how long.

Development: Expertise | Opportunity/Misery

The vagaries of educational administration place middle school math teachers in a precarious position. This tension is between building the experience base necessary to become an expert teacher and pursuing career opportunities and/or avoiding professional misery. Movement can require gaining new skills, which takes time, and stunts gains in familiarity—i.e., with students, colleagues, and technologies. This aspect shapes decisions about how to manage one's long-term development.

Three emergent features of context

Three emergent features of the context of teaching work create challenges for teachers to navigate.

Simultaneity. To set up their framework of differentiated skills inherent in teaching, Geel et al. (2018) offered a view of "four *chronological* differentiation stages can be distinguished that are closely interrelated: A teacher prepares a lesson (Stage 2) based on the evaluation of the previous lesson (Stage 4) and based on his preparation of the lesson period (Stage 1). This preparation enables the teacher to adequately address the differences between students during the lesson (Stage 3)" (emphasis added). Their representation offers a useful framework and a general "chronology" of tasks that is no doubt widely shared as a mental model of how teaching happens.

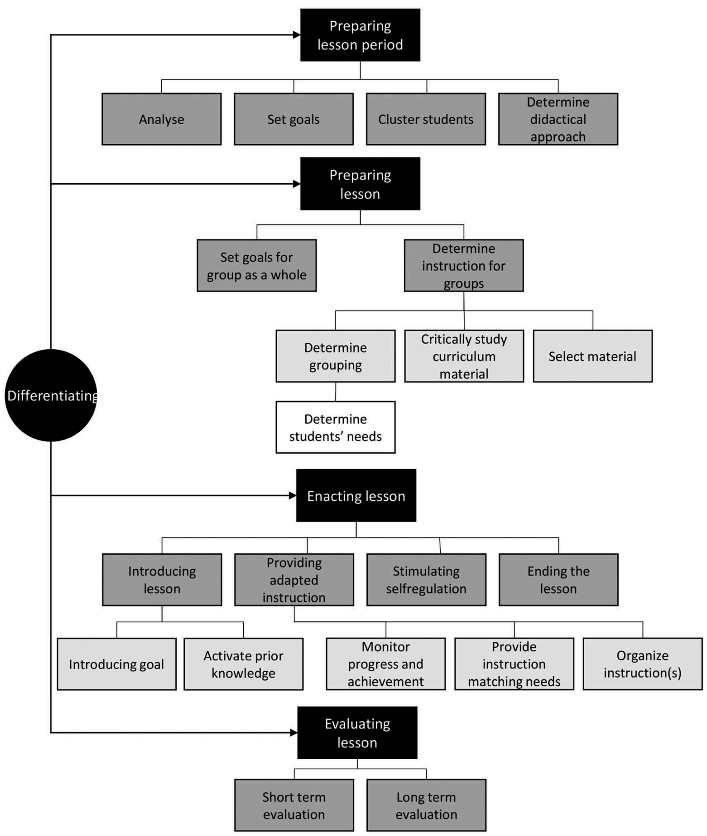

Figure 3: Task flow diagram from Geel et al., 2018

Such representations, however, mask the *simultaneity* of task execution that actual teaching requires. The notion of chronological sequencing of the tasks of teaching offers only limited insight. In contrast, Meyer (2023) offers a powerful illustration of the simultaneity of tasks by analyzing a 45-second video of math instruction during which a teacher bounces effortlessly between clustering students by ability, determining students' needs, and providing in-the-moment evaluation. This example is much more indicative of the simultaneity of task execution that happens in teaching middle school math.

Disruptions. An emerging 'interruption science' has revealed many ways in which disruptions can affect performance. In teaching, these can include interruptions to daily or weekly schedules, distractions such as devices and behavior, and intrusions such as technology implementations, curriculum and faculty changes. They range from the routine and pedestrian – like fire drills and assemblies – to the unexpected and traumatic – like the loss of an advocate. Kraft and Monti-Nussbaum (2021) estimated that "a typical classroom is interrupted more than 2,000 times per year and that these interruptions and the disruptions they cause result in the loss of between 10 and 20 days of instructional time". Of course, disruptions not only affect teachers and teaching, but also the targets of their task execution – the students. In particular, absenteeism presents perhaps the most challenging disruption, as effective teaching is predicated on attendance.

Variability. It is a truism to say that no two students are alike. Even as standards of learning and instruction generally treat all learners the same, save for some differences in learning capacities, teachers face extreme levels of diversity in their classes. Variability exists across students and emerges across time. Other components of the work environment present even more variability, including the quality and comprehensiveness of instructional materials, and the availability, quality, and understandability of performance data. In many domains, controlling for variability enables heightened levels of performance. Teachers have no such luxury.

Within these contexts, middle school math teachers attempt to execute the tasks that comprise teaching.

The cognitive tasks teachers attempt to execute to do the work

The practice of teaching middle school math comprises an extensive set of interwoven cognitive tasks. Any attempt to catalogue them is necessarily reductive, but framing them as cognitive activities brings into high relief *how teachers think* through the work they need to achieve.

The CTA findings revealed four major cognitive tasks, each comprising several subtasks.

- **Assessing** is about determining where learners are in their development. Assessing primarily targets learners as people, their prior knowledge, and their mastery of mathematics.

- **Implementing** is about transforming learning content, tools and resources into learning experiences, including lessons and interventions.

- **Monitoring** is about determining whether learners are engaged and progressing. Whereas assessing is about the moment, monitoring is about the journey.

- **Planning** is the cognitive task of considering courses of action and preparing to execute them in the near- and long-terms.

- **Managing** comprises a set of cognitive tasks for planning for learning, collaborating with colleagues, steering the development of one's self, and dealing with other responsible parties.

Executing each of these cognitive tasks involves facing difficulties, considering available information about the emergent situation, and attempting to complete them using strategies or "moves." While many of the difficulties are ever-present, as teachers gain experience, they develop greater sensitivities to additional difficulties and to situational information that can inform more effective and efficient, workable strategies. And they figure out which of those strategies are most successful.

The following task analysis offers a view of the task difficulties, key information sources, and strategies used by Novice / Advanced Beginner- (N/AB), Competent-, Proficient- and Expert-level middle school math teachers, for each of these cognitive tasks. We derived the N/AB findings from the interviews with AB-level teachers and reflections of higher-level performers about their own early career performance and observations of new teachers.

Figure 4 presents a hierarchy of the cognitive tasks inherent in middle school math teaching.

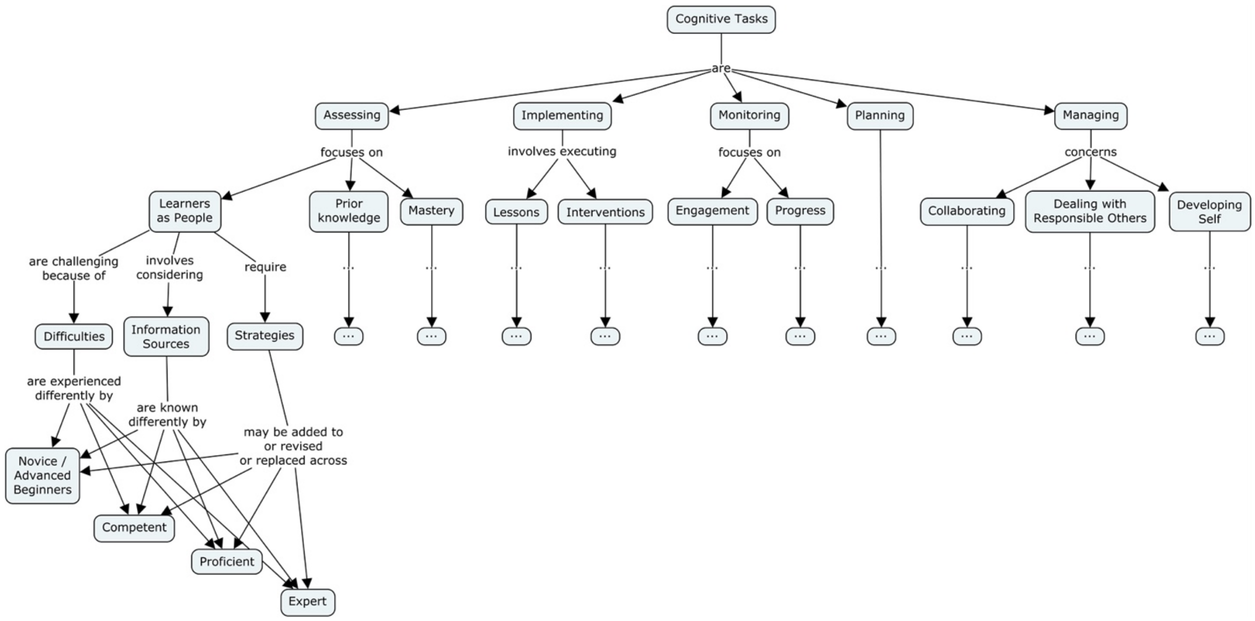

Figure 4: Hierarchy of Cognitive Tasks

For the task analysis findings, it should be assumed that performance at the higher proficiency levels mostly *incorporates* lower levels. That is, performers at higher levels of proficiency face many of the difficulties of lower levels, but may also *experience* them differently or may not characterize such aspects of the work as difficult. They may use the same information sources and strategies of lower levels *when they deem them appropriate*, but also *add to, revise,* and *replace* or *jettison* them.

Where difficulties, information sources, and strategies may cite the same themes but show such variance at higher levels, they are stated with a qualifier in the form "N:N".

To facilitate ease of review, we provide the entire set of cognitive task analysis findings in Excel spreadsheet here:

https://perigeantechnologies.com/publications/CTAofMiddleSchoolMathTeaching.htm.

The mastery of the work they achieve over time

A Mastery Model shows the progression of proficiency as people acquire experience in a given domain, and documents performance progress across levels of mastery. Table 2 above provides a generic Mastery Model of the five stages of progression. Tables 7 and 8 below provide the two components of a Mastery Model for middle school math teaching – stage profiles and performance indicators.

The Mastery Model can be considered an extrapolation of the CTA findings that enables an abstracted view of performance as seen across the levels of proficiency. The value of a Mastery Model lies in describing performance in ways that provide points of comparison. Such points are useful in considering the levels of performance that interventions—including professional development and technologies—*should be expected to affect.* For example, a professional development activity may be classified as targeting particular characteristics and features of performance *to be advanced* from one level to another, with the Mastery Model guiding the intended outcome state. Likewise, technologies may be posited to better / more efficiently / more cheaply enable specific task performance at particular proficiency levels, or even to promote performance to higher levels.

Mastery Models describe five levels of proficiency, starting with Novice. It is important to note that our model merges the Novice and Advanced Beginner stages. We did not conduct CTA with true novices, as all of our participants had at least one year of teaching experience. Moreover, many teachers get classroom experience through student teaching rotations in their academic courses. Thus, true novices in the paid teaching workforce are difficult to find. Future explorations should investigate true novices, including those performers who may have teaching experience but are at a Novice-level for *teaching math*. Indeed, while we did not explicitly investigate differences between teachers who begin careers with deep math content knowledge (the majority of our participants did) and those who do not (as has increasingly become the case in light of teacher shortages), such differences should be the focus of future work.

It is important to note that the levels of proficiency are painted with broad strokes. That is to say that while they are intended to accurately and generically reflect performers and performance at each stage, any individual performer may also reflect some aspects of performance at higher levels. This is to be expected as performers advance. The Mastery Model should not be considered a stage-gate developmental model whereby performers *must* experience and/or achieve *all* noted aspects of the model to 'level up.' Rather, as they gain experience and take actions in response, they will achieve mastery in line with a greater proportion of the expectations at each level. They will also come to view themselves as having achieved their current status and steadily become more aware of what higher levels actually entail.

This said, our CTA interviews captured many examples of the lived experience of performers 'leveling up,' particularly as more proficient performers reflected on their earlier career selves. The story of one participant's experience in particular – and the benefits that followed – brought this experiential process into high relief.

Appendix A presents her story
and should be reviewed as context for the Mastery Model.

Stage Profile for KPAs

Table 7 provides the Stage Profiles for Key Performance Areas (KPAs). These provide a depiction of performer characteristics associated with each of the five stages of learning within each key performance area – in this case, we align the KPAs with the tasks described in the task analysis.

Stage profiles describe "who" performers are and "what they look like" at each stage, drawing in part from the difficulties they note and the information sources and strategies they use.

Table 7: Stage Profiles for KPAs

KPAs	Novice / Advanced Beginner	Competent	Proficient	Expert
	Overwhelmed and lost **Survives**	**Establishes footholds in the face of complexity** **Delivers**	**Covers all the bases with grace and efficiency** **Adapts**	**Goes beyond and just sees things differently** **Innovates**
Assessing	Views students as mysterious or imbued with perceived individual traits – e.g., bad behavior + low confidence = struggle with math	Begins to consider contextual information about learners and their performance Begins to recognize learning gaps and common misconceptions Begins to differentiate between High/Low abilities Includes social and system issues in diagnostic thinking	Able to place observed performance in broader learning contexts, including expected standards Actively seeks and analyzes data from multiple sources Able to routinely recognize misconceptions, learning gaps and patterns of performance based on past and expected experience	Views all learners as capable, with heightened sensitivity to structural challenges to success

Implementing	Views students as bucket model, recipients of content Experiences curriculum as overwhelming set of materials to be delivered to students Requires practice Struggles to adapt to complexity in the moment Views the learning unit as individual students	Views students primarily as calculators Begins to feel ownership of curriculum Able to implement without supervision and recognize features of professional presentation Recognizes potential for complexity Views the learning unit as divisible into subgroups, with focus to be primarily placed on low performers	Views students also as problems solvers Takes ownership of curriculum Recognizes the emergence of complexity and exceptions, and adapts accordingly Views the learning as divisible into subgroups, with focus to be placed also on high-performers and learners in roles	Views students as both learners *and* teachers Owns all aspects of implementation and helps others build ownership, including learners Anticipates complexity and uses principles to guide approaches Views the learning unit as empowered, self-interested and perpetuating

Monitoring	Views the learning unit as needing to be ordered Assumes progress as mostly linear for all students Overwhelmed by data	Views the learning unit as requiring engagement strategies Begins to recognize nonuniformity of progress across students	Views the learning unit as developmentally age appropriate Anticipates varying rates of progress	Views the learner as responsible for gauging their own progress Recognizes structural constraints to progress

Planning	Assumes curriculum is given, current, immutable, and unit focused Attempts to paces according to predetermined structure Experiences many surprises from envisioned plan and execution misalignment	Begins to experience constraints on the curriculum as designed Begins to plan beyond curriculum, to environment Begins to see units as interconnected	Understands the relationship of curriculum to broader standards of learning Understands the interconnectivity of units Appreciates variability of curriculum quality and systemic causes	Assumes adaptive approach Understands anterior and posterior standards of learning across grade levels

Managing	Engagement with others is viewed as demanding and primarily focused on corrective feedback on own performance Sense of self as awkward and unprepared – in content knowledge and/or pedagogy Desire to please, connect, and improve	Lingering doubts about performance and requirements for interaction with others Understands limitations of preparation and appreciates structural challenges Begins to take ownership of self-development Emerging self-awareness about potentially successful approaches, including vulnerability	Actively pursues and manages relationships, especially challenging ones Appreciation for and pursuit of continuing professional development Comfort with uncertainty and trust in self	Sets expectations for others Places feedback and critiques in broader contexts, and responds accordingly

Performance Indicators

Table 8 provides the Performance Indicators for each KPA, at each stage. These show the actions, attitudes, or behaviors an individual is likely to exhibit at each stage.

Performance Indicators describe "how" performers go about their work, drawing in part on the information sources and strategies they use.

Table 8: Performance Indicators

KPA	Novice / Advanced Beginner	Competent	Proficient	Expert
Assessing	Places focus on what individual students do Primarily reviews performance data from completed work and direct observations of behavior	Primary places focus on what individual students do and how are they doing it; begins to consider group performance Uses a variety of assessment practices to assess and categorize individual learners, including in-the-moment and post-performance techniques and data organization and exploration	Primarily places focus on what all students are doing, and how their performance compares Uses a broader variety of assessment practices, including innovations, extensive data analytics, and comparative analyses, to target specific needs of subsets while maintaining continuous evaluation of the entire group	Actively enables and reinforces student data awareness and implications Places focus on what all students are capable of Uses complex resources and activities to enable learners to demonstrate mastery Evaluates learning requirements and strategies of subgroups

Implementing	Places primary focus on singular and predominantly algorithmic outcomes Executes 'scripts' and mostly simple, individual activities Expresses uncertainty about when and how to intervene Limited repertoire of instructional techniques Delivery appears inauthentic and lacks confidence	Places primary focus on conveying (plural) algorithms and strategies, finding and correcting misconceptions, and secondary emphasis on conceptual Executes increasingly complex activities, including some group-based Introduces interventions based on noted errors Expanded repertoire of instructional techniques, primarily focused on the individual learner Begins to build relatable connections to content	Places increasing focus on conceptual aspects Executes complex activities, including social interactions and innovations, targeting all levels of learners and across a variety of instructional contexts Considers potential trade-off value of interventions and effort Actively exploits intervention opportunities in a timely manner	Places focus where appropriate – i.e., practicing, procedures – given content and student achievement Executes activities beyond constraints, including classroom space and time Interweaves lessons through applied projects Execution is highly structured yet allows for exploration Enables purposeful opportunities for students to identify alternative approaches, gain efficiencies, and self-report errors Delivery appears natural, even entertaining

Monitoring	Places high priority on behavioral data Default to simple data reporting Focuses on individual learner progression Performance derailed by interruption	Places priority on behavioral data while expanding consideration of performance data Gauges individual progress in the context of class Uses divide and conquer approaches, with a focus on cusp and lower-level performers	Places priority on performance data, particularly task *execution* Gauges individual and group performance in broader contexts Uses divide and conquer approaches, with strategies for high-level performers	Places priority on performance data, particularly task *interest* Enables students to build meta-cognitive skills at evaluation Shows graceful recovery from interruptions Gauges individual and group performance by novel indicators of capability achievement Communicates progress

Planning	Receives curriculum Prepares on the fly with significant effort Primarily focuses on the immediate requirements	Begins developing and/or refining some curriculum elements drawing on prior execution Primarily focuses on the upcoming requirements	Engages in extensive curriculum development and/or refinement drawing on broader feedback and standards Includes potential student responses, and mixes concrete and abstract elements, in materials and activities	Considers cross-grade-level alignment Paces with flexibility Plans year-round and collaboratively

Managing	Attends meetings and receives feedback Attempts to engage Responsible Others (ROs) through discussion Observes and follows other teachers	Actively engages in self-improvement activities, both with others and alone Builds multiple channels for engagement with ROs	Extends collaborative information exchange across many platforms Engages ROs routinely for multiple purposes Self-evaluates using robust rubrics	Inspires others to advance and actively enables their improvement Conducts extensive and ongoing self-evaluation, and transforms performance accordingly

The macrocognitive processes that experienced teachers use to achieve the work

A macrocognitive analysis consolidates findings about experienced domain practitioners along the known macrocognitive dimensions of performance (Klein et al., 2006). This view on performance is useful for further specifying the aspects of performance that differentiate proficient and expert performers from the rest and characterize what expertise looks like so that interventions designed to accelerate and/or enable higher-levels of performance have a target.

Table 9 offers a review of the macrocognitive dimensions and known capabilities of high-level performers.

Table 9: Macrocognitive Dimensions and Capabilities

Macrocognitive Dimension & Key Source	Known Capabilities of High-Level Performers
Naturalistic (Recognition-Primed) Decision Making (Klein, 2017)	Diagnose situations through recognizing typicality and discriminating key features; Mentally simulate workable course of action; Act quickly
Sensemaking (Klein, Moon, Hoffman, 2006)	Refined mental models about how things work help to select data that matters; Data helps select the best frame; Explain
Flexecuting (Klein, 2007)	Set and revise goals based on discoveries made during execution
Projecting & Anticipating (Klein, Snowden & Pin, 2011)	Prepare for future events by anticipating trajectories
Keeping the Big Picture (Hutton & Klein, 1999)	See antecedents and consequences; Understand the import of contexts
Common Ground (Klein et al., 2005a)	Manage co-responsible tasks across time and maintaining mutual beliefs and awareness among stakeholders
Managing Attention (Woods, 1995)	Actively direct apparati to monitor relevant information
Managing Uncertainty (Klein, 2004)	Monitor and respond to ill-defined contexts
Detecting problems (Klein et al., 2005b)	Develop sensitivity to emerging situations; See and represent problems at a deeper level; Spend ample time trying to understand the problem
Improvise (Klein, 2013)	Do more with available resources; Find novel solutions
Motivation (Moon & Bildstein, 2019)	Active recognition of performance and response to feedback, fostering self-development

Our CTA findings from Proficient- and Expert-level teachers revealed examples of these hallmark macrocognitive dimensions across all of the cognitive tasks, as shown in Table 10.

Table 10: Macrocognitive Dimensions of Middle School Math Teaching

Macrocognitive Dimension	Exemplars in middle school math teaching
Naturalistic (Recognition-Primed) Decision Making	Experience informs how to implement lessons in ways that are most likely to be successful for the majority of learners to achieve conceptual and procedural success, assess mastery through a variety of data points available in the classroom environment, and call upon an extensive repertoire of interventions to help learners advance.
Sensemaking	Drawing on a large and diverse set of frames about how learners deal with particular content at particular stages of development, primes the abilities to see misconceptions at a glance, diagnose content gaps, categorize learners and their circumstances, select and evaluate specific data about student performance, and know which frames are appropriate to guide understanding, given the available or missing data about learners.
Flexecuting	Many repeated cycles of planning and execution under continuously changing contexts of administrative leadership, colleague rotation, standards evolution, interruptions, and curriculum revisions, enables adaptive planning, including workable pacing and sequencing, towards the never-changing goal of learner improvement.
Projecting & Anticipating	Engagement with hundreds of learners and their Responsible Others across the full-spectrum of academic calendars enables rapid assessment of learners' starting points and their *potential* trajectories, which in turn informs which interventions may be necessary to ensure pivots toward *desired* trajectories.
Keeping the Big Picture	Deep understanding of the educational apparatus, in particular the expected standards of learning across all grade levels, provides the context for understanding whether learner progress is tracking appropriately and devising plans that set up learners for future success.
Maintaining Common Ground	Routine and ad hoc collaborative activities, including robust information sharing across the network of stakeholders (i.e., learners, teachers, and Responsible Others), provides continuous assessment of mental models about learner status, in turn supporting timely implementation of necessary interventions.
Managing Attention	Long-established and reinforced routines, targeted inquiries, seamless facilitation, and novel and distributed data collection help to maintain awareness across the large, complex unit that is a class, and in turn, enable learners to keep focus on the tasks at hand.
Managing Uncertainty	Hard-earned trust in one's teaching process and abilities, including skills at relationship building, engenders a patient approach to gaining familiarity where uncertainty exists, most notably in dealing with a new class of learners.
Detecting problems	Thorough, timely and collaborative dives into learner and teacher performance data provide early insights about which strategies may not be working and which learners need help.
Improvise	Having faced the full spectrum of constraints requires devising creative uses of resources, seeking leverage points to do more with less, and maximizing the learning value of projects.
Motivation	Years of reflection on practice and diversity of feedback encourage the drive toward achieving and sustaining high-level performance.

The difficulties of subdomains

The CTA findings provide a set of difficulties faced for each of the cognitive tasks, as experienced at the different levels of proficiency.

In addition to these, we also sought to capture a broader sense of the difficulty of math domains *as perceived by teachers*. Specifically, we asked participants to rank order the domains *within grade levels they have taught*, based on their perceptions of the difficulty in teaching the domain *and* the difficulties they perceive students having. Thus, topics perceived to be the most difficult for a given grade level were placed first, next most difficult second, and so on.

We used Achieve the Core's Coherence Map for the set of domains and their descriptions.

Perceived Difficulty Ratings by Domain

Ordered from left to right by *decreasing* difficulty, Tables 11 – 14 provide the perceived difficulty ratings by domain offered by our participants.

Table 11: Perceived Difficulty Ratings by Domain, Grade 5

5	Number and Operations - Fractions	Operations and Algebraic Thinking	Measurement and Data	Geometry	Number and Operations in Base Ten
	Use equivalent fractions as a strategy to add and subtract fractions. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	Write and interpret numerical expressions. Analyze patterns and relationships.	Convert like measurement units within a given measurement system. Represent and interpret data. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	Graph points on the coordinate plane to solve real-world and mathematical problems. Classify two-dimensional figures into categories based on their properties.	Understand the place value system. Perform operations with multi-digit whole numbers and with decimals to hundredths.
Mean	1.18	3	3.3	3.44	3.73
Median	1	3	3	4	4
Mode	1	2	3	5	4

Table 12: Perceived Difficulty Ratings by Domain, Grade 6

6	Ratios and Proportional Relationships	Expressions and Equations	The Number System	Statistics and Probability	Geometry
	Understand ratio concepts and use ratio reasoning to solve problems.	Apply and extend previous understandings of arithmetic to algebraic expressions. Reason about and solve one-variable equations and inequalities. Represent and analyze quantitative relationships between dependent and independent variables.	Apply and extend previous understandings of multiplication and division to divide fractions by fractions. Compute fluently with multi-digit numbers and find common factors and multiples. Apply and extend previous understandings of numbers to the system of rational numbers.	Develop understanding of statistical variability. Summarize and describe distributions.	Solve real-world and mathematical problems involving area, surface area, and volume.
Mean	2.64	2.86	2.86	3.14	3.50
Median	2.5	2	2.5	3	4
Mode	1	2	2	3	4

Table 13: Perceived Difficulty Ratings by Domain, Grade 7

7	Ratios and Proportional Relationships	Statistics and Probability	Expressions and Equations	Geometry	The Number System
	Analyze proportional relationships and use them to solve real-world and mathematical problems.	Use random sampling to draw inferences about a population. Draw informal comparative inferences about two populations. Investigate chance processes and develop, use, and evaluate probability models.	Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	Draw, construct, and describe geometrical figures and describe the relationships between them. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
Mean	2.43	2.93	3.07	3.14	3.43
Median	2	3	3.5	3	4
Mode	2	1	5	3	4

Table 14: Perceived Difficulty Ratings by Domain, Grade 8

8	Expressions and Equations	Functions	Geometry	Statistics and Probability	The Number System
	Work with radicals and integer exponents. Understand the connections between proportional relationships, lines, and linear equations. Analyze and solve linear equations and pairs of simultaneous linear equations.	Define, evaluate, and compare functions. Use functions to model relationships between quantities.	Understand congruence and similarity using physical models, transparencies, or geometry software. Understand and apply the Pythagorean theorem. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	Investigate patterns of association in bivariate data.	Know that there are numbers that are not rational, and approximate them by rational numbers.
Mean	2.37	2.63	2.74	3.11	4.16
Median	2	2	3	3	4
Mode	1	2	3	4	5

Perceived Difficulty Ratings by Proficiency

Given that performers at different proficiency levels experience different difficulties and develop information sources and strategies to deal with them, it stands to reason that teachers' perceptions of difficulty for the domains would differ across proficiency levels.

Figures 5 through 8 present the trend analyses of the perceived difficulty ratings across the proficiency levels. Where trends are particularly pronounced, they are shown in block lines; less pronounced trends are show in dotted lines.

Appendix B includes a more detailed view of the perceived difficulties by experience and a set of strategies that teachers use to encourage learning in each domain.

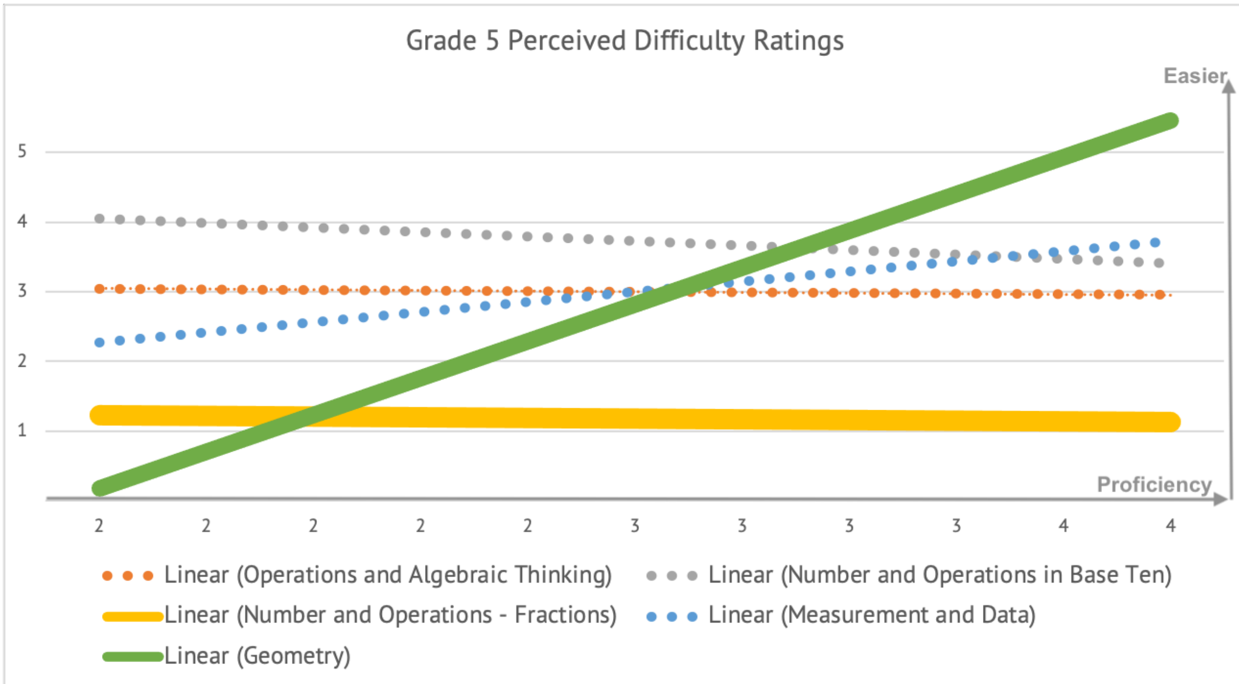

Figure 5: Grade 5 Perceived Difficulty Ratings, by Proficiency Level

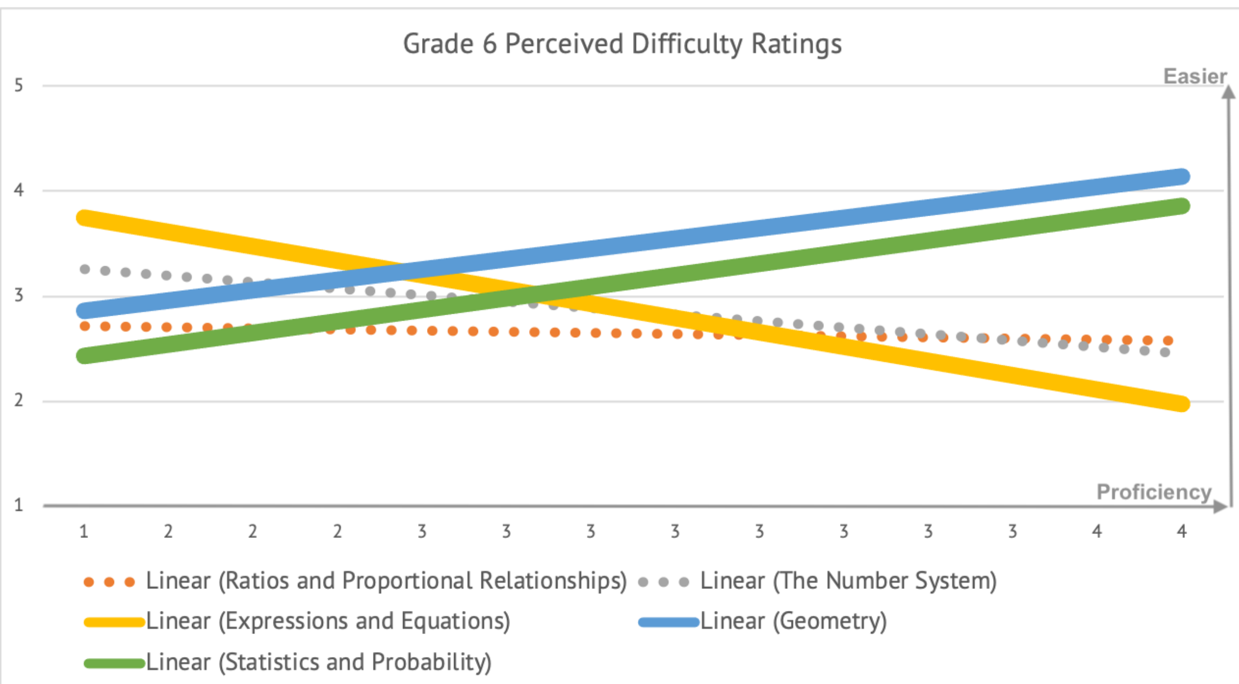

Figure 6: Grade 6 Perceived Difficulty Ratings, by Proficiency Level

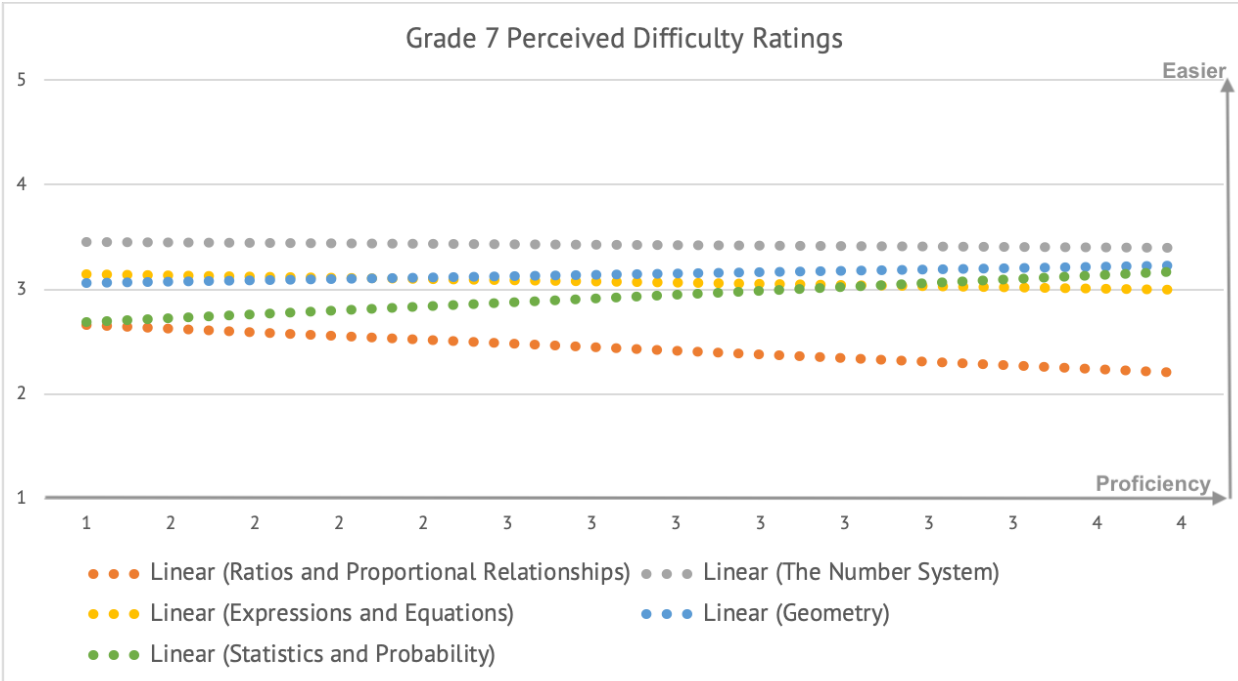

Figure 7: Grade 7 Perceived Difficulty Ratings, by Proficiency Level

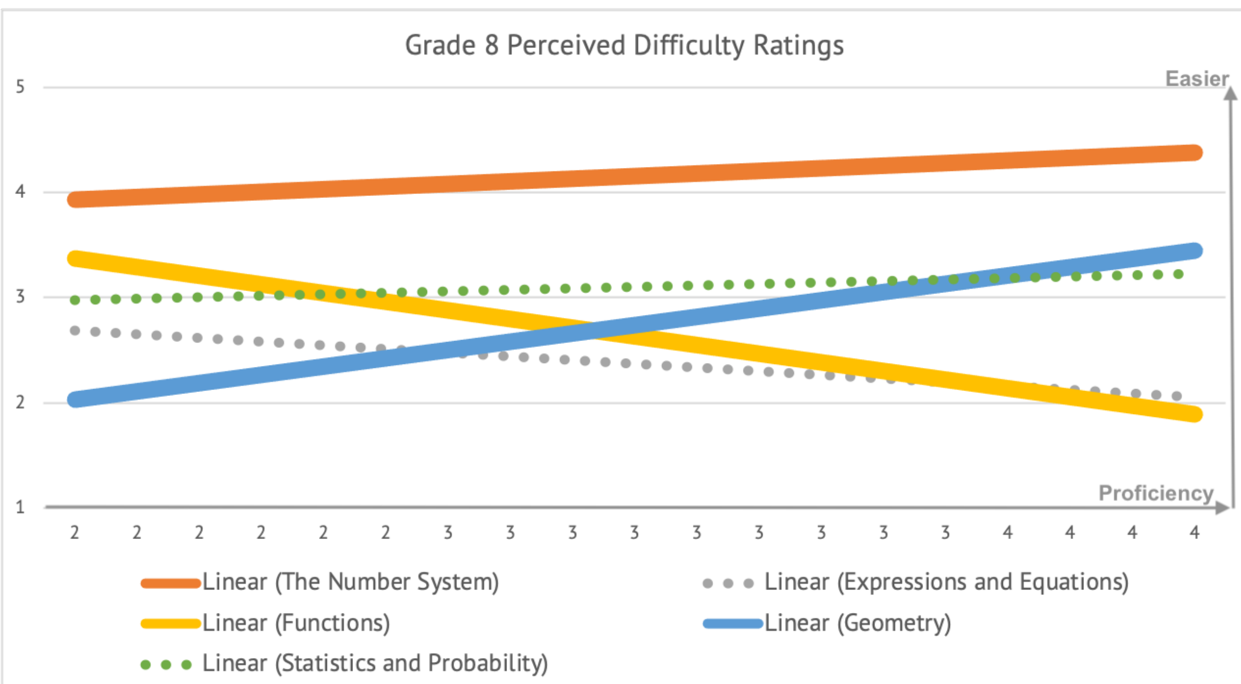

Figure 8: Grade 8 Perceived Difficulty Ratings, by Proficiency Level

Recommendations

While the primary goal of this CTA of Middle School Math Teachers is to describe cognitive performance, we also derived some recommendations for accelerating proficiency achievement and for designing and evaluating technology enablers, specifically computing technologies.

These recommendations focus squarely on enabling teacher performance to more quickly reach the Proficient and Expert-levels of mastery described above, and augmenting their performance through technological innovation, while also helping teachers mitigate some of the challenges in using technologies that have been observed in other domains.

Most importantly, these recommendations should be implementable in the context of educational operations as described by our participants – i.e., they do not call for wholesale re-engineering of the educational system.

Extending the Mastery Model

The first set of recommendations offers directions for extending the Mastery Model, and they are informed by the limitations to the current Mastery Model, as noted above.

Future work toward understanding the cognitive performance of middle school math teaching should focus on extending the Mastery Model to include, at least:

- investigate differences between teachers who begin careers with deep mathematic content knowledge and those who do not,
- investigate true novices, including those performers who may have teaching experience but are at a Novice-level for *teaching math*,
- additional Expert-level findings,
- deeper dives into the subdomains perceived as most difficult,
- findings from *in situ* observations.

Creating the Conditions for Expertise Development

The next set of recommendations offer pathways for ensuring that expertise can develop, and for hastening its arrival. They are informed by the CTA findings and the known general conditions for accelerating performance toward mastery (see for example, Hoffman et al., 2013; Klein and Baxter, 2006; Moon and Bildstein, 2019).

Known General Conditions for Expertise Development

We know what is required for performers to achieve professional mastery. Table 13 shows the key enablers and disablers of expertise development.

Table 15: Known General Conditions for Expertise Development

Enablers	Disablers
Compiling an extensive experience bank • Especially edge cases that stretch knowledge and skill	Stunting experience bank • *Only* having access to basic problems and/or incomplete access to the total problem
Managing motivation • Two-way street marked by attempts and feedback	Discouraging motivation • Failure to submit to expertise • Lack of expert(ise) available
Encouraging perspective shifts • Role, self, purpose	Perspective stagnation • Entrenching in mindsets
Getting the right kind feedback • Accurate, diagnostic, timely	Bad or no feedback • Not accurate or diagnostic or timely or direct
Evolving mental models • Including strategies for tracking, refining, instantiating	Retarding mental models • No active attempts to refine or track
Practicing • Especially with performance-relevant goals and evaluation criteria	Bad or no practice • Goal-less, no evaluation criteria
Fostering metacognitive skills • Especially becoming more mindful of opportunities for learning	Disabling metacognitive skills • Few opportunities for learning or significant reflection

Setting these conditions requires work from both the school system and teachers.

School conditions

School systems can help accelerate the achievement of proficiency primarily by ensuring that teachers gain experience within the contexts of their delivering on their responsibilities. Such demands are no doubt difficult to meet given staffing constraints, but the implications of our findings suggest the following approaches would be advantageous.

Maximize familiarity. The cognitive tasks of Assessing and Monitoring are best enabled through building deep familiarity with students over time. The critical importance for Experts of deep familiarity with students was seen decades ago by Berliner, when he attempted to isolate the *teaching* from the students:

> Teachers were given 30 minutes to plan the lesson. While they taught, they were videotaped, and after the lesson, during stimulated recall, they were asked to tell us about their thinking and justify their actions during teaching. Despite the fact that the experts performing this task were judged to be better teachers on a number of dimensions, *the task triggered a good deal of anger among them. One of them quit the study, another broke down and cried in the middle of the study, and all were unhappy they participated.* They all reported their fears about performing well when we moved them from their own classrooms to the laboratory situation we had created for them to teach in (Berliner, 2004; p. 202; emphasis added).

Schools must seek approaches for maximizing and preserve engagement between students and teachers. Such strategies should include direct approaches, such as looping when feasible, and indirect approaches, such as teacher-to-teacher handoffs, which offer significant opportunities for knowledge continuity but are rarely developed in an active way (see Moon and Hayden, 2022).

Assign new teachers to the middle grades. Many students enter middle school with a variety of gaps from elementary school, and preparing students for high school mathematics requires knowledge of the path ahead. Also, the perceived difficulty ratings suggest that grade 7 content is the least difficult to teach. It stands to reason, then, that assigning new teachers to teach grades 6 or 7 would enable them to gain experience while being buffered by more experience teachers.

Take a resiliency-engineering stance. Two major themes from the CTA findings concerned the variety of attempts to design systems that meet all needs, for all learners, all the time – and the variety of ways in which such systems were confronted with surprise. Pacing and sequencing in curriculum planning is but one example. "Resilience engineering is a subfield of safety science research that focuses on understanding how complex adaptive systems cope when encountering a surprise. The term *resilience* in this context refers to the capabilities that a system must possess in order to deal effectively with unanticipated events. Resilience engineering examines how systems build, sustain, degrade, and lose these capabilities" (Wikipedia, 2023). Middle school math teaching is a bastion of a complex adaptive system – this recommendation promotes building a resilience engineering perspective into administrative roles.

Professional development

The professional development apparatus can help teachers achieve mastery primarily by enabling them to rapidly gain experience with the most challenging cases – i.e., students, classrooms, schools – and receive the right kind of feedback as they attempt teaching.

Case-Based Learning Experiences (CBLE). While experience is necessary for expertise development, not all experience is the same for helping to advance toward mastery. Professional development in the form of CBLE offers the best opportunity to rapidly acquire a broader experience base with complex problems, to get the right kinds of feedback, to practice, and to evolve one's mental models. This recommendation brings a long pedigree. In 2000, Putnam and Borko called for the use of CBLE, noting:

> Rather than putting teachers in particular classroom settings, cases provide *vicarious* encounters with those settings. This experience of the setting may afford reflection and critical analysis that is not possible when acting in the setting (Putnam and Borko, 2000; p. 8).

They also noted the lack of research regarding the approach, at the time:

> Despite vocal advocates and an increased use of cases in recent years there is much to learn about their effectiveness as instructional tools (p. 8).

Twenty-plus years on, we now have a great deal of evidence demonstrating the benefits of CBLE, specifically the value of using challenging cases to carve a path toward cognitive transformation and revising mental models (see Hoffman et al., 2013). Moreover, we have evidenced-based practice informing the design of such learning experiences, including applications of augmented reality (see Militello et al., 2023), though even low fidelity CBLE can induce valuable learning, provided the cognitive fidelity is high. Such experiences could be designed to provide new and even experienced teachers with cases that stretch knowledge and skill and give access to a spectrum of complex problems in teaching middle school math. It is important to note that the starting point of all efficacious CBLE is the development of rich scenarios – a task to which CTA is particularly well-suited. Such scenarios could elicit, for example, the practice of skills for:

- Dealing with interruptions and curricula pivots,
- Managing challenging ROs,
- Engaging learners at both ends of achievement
- Co-teaching,
- Innovating Lessons and Interventions,
- Recognizing subtle cues in performance,
- Engendering relatability,
- Giving and responding to feedback,
- Teaching "new to me" grades and domains.

Practice. While CBLE offers the potential to cover a lot of ground, developing some skills required for proficient teaching of math requires practice through repetition, which is difficult to come by in environments already strained for time. These include building content knowledge and identifying misconceptions. The use of artificial intelligence offers the opportunity to refine these skills. For example, teachers could attempt problems in the curriculum themselves and be shown multiple approaches for problem solving to expand their repertoire of solutions and approaches. Teachers might also assign problems to AI-powered "students," which could in turn generat responses that embed known misconceptions to help teachers hone their recognition skills.

Use Mastery Model as a rubric. Given that the Mastery Model describes proficiency levels, it is well suited to serve as a rubric for evaluating where teachers are along the pathway toward proficiency. Administrators, personnel in coaching roles, and even new teachers should, at the very least, be familiar with the Mastery Model so that their recognition skills may be attuned during opportunities to observe performance and provide feedback. Proficiency is much easier to spot when one knows what to look for.

Develop mentorship skills. The cognitive processes of Collaborating and Developing Self emphasize the importance of the mentoring relationship. Unfortunately, as education researcher Christine Pfund has shown, teaching follows a familiar pattern seen in other domains: "We're putting our precious trainees in the hands of folks who are well-intentioned but have had no professional development in the arena of mentoring." Mentorship is a learned skill that requires instruction, practice, feedback, self-reflection, and intention. This recommendation is for the inclusion of mentorship training in professional development; specifically, training that provides tools for effective knowledge sharing and motivation building. Such training can also help mentors recognize when mentees use "knowledge shields" to protect their ways of thinking – a critical skill for helping them evolve their mental models (Feltovich et al., 1994).

Technology applications

The final set of recommendations offer hypotheses about designing technology applications that enable proficient performance—and help it to develop—yet also do not disable the use of expertise. They are informed by the CTA findings, principles of human-centered computing (HCC; Hoffman, 2012), and the evidence base about how some technologies impede proficient performance (see Klein, 2004; Moon and Hoffman, 2005;).

Principles of HCC and How Technologies Impede Proficient Performance

We know what is required for information technologies to augment human expertise – and also how they can get in the way. For technology designs to support middle school math teaching, three principles of HCC—and the consequences of not heeding them—are particularly relevant, as shown in Table 16:

Table 16: HCC Principles and Consequences

HCC Principles (Hoffman, 2012)	Consequences (Klein, 2004; Moon & Hoffman, 2005)
The Aretha Franklin Principle: Do not devalue the human to justify the machine. Do not criticize the machine to rationalize the human. Advocate the human–machine system to amplify *both*.	Problems arise when (a) the new combination underperforms because the technology interferes with our intuitions; (b) the human + computer combination only outperforms humans in limited settings and otherwise breaks down; or (c) we've lost our intuitive skills by the time we discover the limitations of the technology.
The Sacagawea Principle: Tools need to support active organization of information, active search for information, active exploration of information, reflection on the meaning of information, and evaluation and choice among action sequence alternatives.	• Disrupts Pattern Recognition by Disconnecting Us from the Data – e.g., by automatically providing information rather than letting us work with the data ourselves • Limit How We Search for Data – e.g., by assuming a predefined dataset as essential or nonessential and compiling • Provides More Than We Need – e.g., by pushing data assumed to be relevant even when duplicative, outdated, or nonsensical
The Lewis and Clark Principle: The human user of the guidance needs to be shown the guidance in a way that is organized in terms of their major goals. Information needed for each particular goal should be shown in a meaningful form and should allow the human to directly comprehend the major decisions associated with each goal.	• Weaking Mental Models – e.g., by restricting categories and algorithms from evolving • Hides the Story of How It "Thinks" About the Data – e.g., by not permitting access to its reasoning • Make Us Less Adaptive – e.g,. by requiring us to follow procedures, even in light of changing circumstances • Make Us Passive – e.g., by lulling us into believing the automation is always right

Designing technologies to support middle school math teaching requires consideration of whether and how they may augment or impeded expertise.

Potential augmentations

Data tracking and exploration. The CTA findings revealed many examples of teachers tracking and interrogating data, particularly through 'homegrown' tools like notebooks, spreadsheets and shared drives. Tools that enable malleable data tracking and flexible interrogation might be introduced to aid teachers in tracking, among other things:

- Student performance over time, particularly in grades prior to and posterior to their current grade, and readily shareable across faculty, students and ROs,
- Prior student performance against particular implementations,
- Editable categories for manual inclusion of innovative metrics,
- Own performance, including navigable recordings of performance,
- Opportunities and constraints for engaging with ROs.

Curriculum management. Middle school math teachers face a dizzying array of curriculum representations, portrayed across a variety of tools, and managed by an assortment of personnel with varying perspectives. Even representations from the same vendor can offer inconsistencies. As with data solutions, we found just as many instances of kludges and workarounds created by teachers to understand the big picture and intent of the content. In addition to representing the curriculum and organizing resources, tools that enable curriculum management should consider supporting:

- Tradeoffs in sequencing and pacing,
- Explicit relationships across lessons, units, strands, and grade years,
- Sharable hypotheses about expected student struggles, linked to specific gaps, and observations of student insights,
- Fungibility and extensibility guidance.

Potential impediments

Masking student work. Process data are an essential component to the cognitive task of Assessing. No matter the proficiency level, all math teachers must have direct access to student work. Indeed, the majority of our participants continue to prefer the use of paper and pencil for students to work out and record their thinking – with oral presentation being the equally best option. Technologies that mask student work stand to disable the ability of teachers to assess students in the process of learning.

Opaque inner workings. As information technologies become increasingly complex, it becomes equally as important for them to provide users with understanding of how they work and why they produce the results they do (Hoffman et al., 2018). It is one thing to run calculations faster than a human can; it is quite another to produce judgments and recommendations based on algorithms that only take a partial view into consideration. Technologies that offer only an opaque view into their inner workings stand to disable the ability of teachers to make effective use of their potential.

Bad implementation. The CTA findings revealed many instances of technology vendors offering little support to their intended end-users, enterprise software purchases left unused, and repeated critiques of widely-sold tools that were a poor match to actual need. At best, such examples are a misuse of funds. But they are at their worst with paired with an administrative mandate for their use. Technology implementations that do not follow a learning engineering approach stand to disable teachers' ability to adapt and turning learning teachers into passive tool operators.

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Appendix A: The Story of One Teacher's Leveling Up Experience and the Benefits that Followed

An experienced middle school math teacher, Charity Rock, shared her experience in leveling-up to the Expert-level and the benefits that followed.

I'd been teaching for about a decade in California. I'd always pass by one middle school in our district and think, "Oh my gosh, I'm so happy I don't work there."

Then I was placed there.
Teaching 8th grade algebra.
At Washington Middle School.

We were impoverished. We had homeless students. We were in the middle of a desert of crime. Our whole school's suspension and absentee rates were dire. We had every indicator that our school needed help. Absent students got support, but there wasn't a pre-planned system.

I was so upset to be placed there.

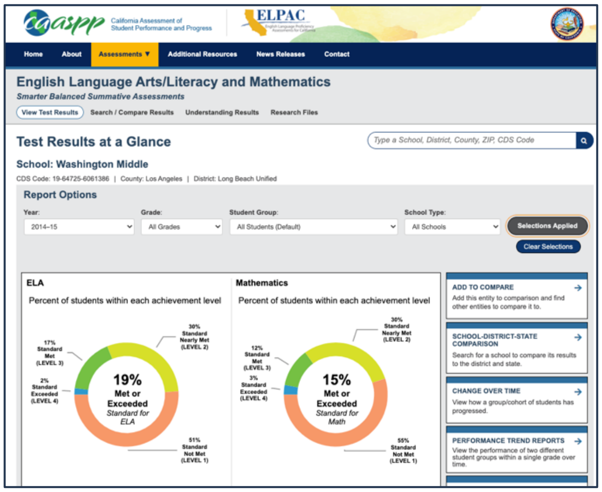
We were the lowest performing school in the district. In my first years, we had district end-of-the-year assessments. Those were common across all schools, but we didn't necessarily see each other's scores. The scores weren't broadcasted. It was very individualized data sharing. I saw class scores and was like, "Okay, my kids are doing fairly well." My students' scores weren't where I wanted them to be, but they definitely weren't terrible.

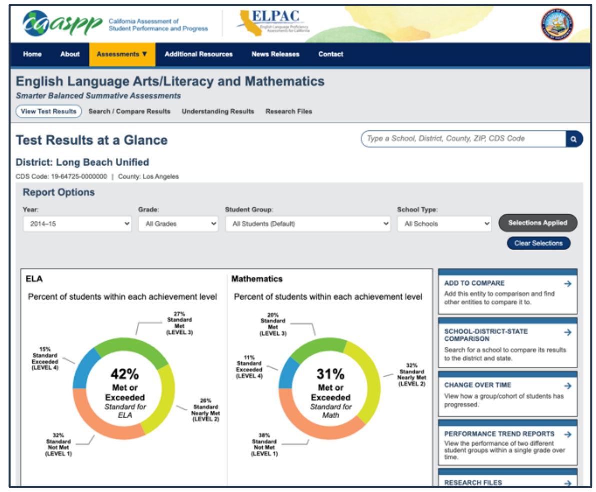
I think I arrived at this very fluke-ish time for assessments. I don't remember getting any jarring numbers. I was in my own private classroom, so I thought everyone was teaching similarly. In our team meetings, I don't even know if I had an agenda. I think perhaps we were talking about the kids. Maybe we were talking about what was coming up next. It started. It ended. Have a good day! I don't remember any time going into planning any of my department meetings, nor do I remember being pressed for time. I don't think it was even that department specific.

We had previously had the Standardized Testing and Reporting (STAR) test. Then we were in a transitional phase, we went maybe two years without a state test. Then, they introduced CAASPP, so it was preliminary, and the scores didn't "count."

But...

Our first year, we were about 15% proficient. We were the lowest performing school in the district. There were a couple super high performing schools. The average was about 30% throughout the district. We didn't get the data until the end of the school year.
I was shocked. I was so sad.





That's when I said:
"Nope." I was like, things have to change.
Business as usual wouldn't cut it.

I had an opportunity to see that students can't study their way into doing well on the CAASPP. They can't memorize every single question. Students must know how to think critically and unpack a question so that every time they see a new one, it makes sense.

Professional Epiphany

I had been a really precise teacher. I was so clear. I could explain anything and make it make sense. I used to get great feedback. I would have visitors come and say: "Wow! You're great! Your kids are engaged." I didn't have too many students absent in my class, so that wasn't a major issue. I can't recall how my students performed, but they weren't as bad as the school's overall 15%.

However...

I received feedback earlier in the first year of CAASPP. It was my principal's first year taking over as the principal. She was accompanied by about four district-level folks to observe the teachers and provide feedback. At first, they debriefed us. Then, whoever's classes they visited, we would go in and get feedback from the group. It was verbal, but I'm sure they wrote things down somewhere. I was ready to receive all the good laurels because I was teaching the heck out of it. I would ask myself a question, then I would answer my whole question. I was just on it. I was on such a high because I thought I was about to get this good feedback again. I'm sure I had a book that I read about CAASPP, but I was like: "I'm already doing this. I'm good to go."

Then they told me I talked too much
and didn't let the students think for themselves.

I was stealing all the thinking from my students. The feedback was: “You explained it great, but the students didn’t say a thing.” You’re still teaching the old-school way. Some students understood the topics because I explained things so well, but I didn’t give them enough time to think about the problems, make mistakes, fix their mistakes, and collaborate. I didn’t offer that space in my classroom.

I don’t even know how I responded.
I know I was shocked, which has stuck with me.
I had never received negative feedback before.

I was known as a wonderful teacher, but I respected the group’s feedback, and I respected my principal. They had been trained on the new CAASPP standards, what it entails, and all the questioning involved. They were all in agreement about my feedback. I couldn’t deny what they said.

Talking too much was the feedback that really shifted things for me. I’m so grateful for it, but in the moment, I was like: “What the heck?”

I was pumped up. I turned that sadness into pure energy, like: “We got this. Let’s just do it.” I started making shifts on my own. No one else necessarily did.

This was me developing into the teacher I am now.

Student learning focus

For me, the biggest piece was stepping out of the way of the students. The secret formula is: “Get out of the way, ask questions, and let the students experience it.” That’s it. They’ll get it. I put big signs on my board to remind me and the students of this.

One sign said: “Who is doing the thinking?”

That’s a powerful statement and question. I think about that as a teacher in *everything* I do—every plan I make, every implementation of the plan.

Another sign said: “Not the fast answer. The thoughtful answer.”

For that to really unfold, the teacher must facilitate the class in such a way that we are saying, “Slow down. Everyone has two minutes of think time. Jot down what you’re thinking.” Those small implementations allow students to access the information before someone says the answer. The shifts in our lesson planning, co-teaching, looking at assessment data, and teaching practices promoted protected, independent think time in all classrooms. Everyone had an opportunity. We were also looking for students to jot down what they are thinking, so when it’s time to turn and talk, they have something to say, and they have something tangible to refer back to. It’s also important to have opportunities for students to revise their work.

Regardless of the activity, the most important thing is protected, independent think time. I was conscientious of how much time students needed to work independently and what I was doing when they were working independently. I was circulating and jotting down what kids were struggling with, the different answers, the different strategies, and the mistakes. Then, I grouped kids based on their work. Some teachers think independent think time is for after you teach them. That's practice time. You want them actively thinking throughout the entire lesson.

The biggest thing is teaching students that
individual thinking is honored and protected,
regardless of what type of project they are working on.

Even when students took notes, I had them fill in notes, so they weren't wasting time copying stuff down. There were so many questions posed in the notes, like "What do you think you should do next? What does this mean?" I gave them 30 seconds to think and 30 seconds to talk to their neighbor.

Kids were thinking for themselves, talking about it, and revising it.
I never just told them what to do and what to think.

Classroom Environment

I didn't have any disruptions. I knew what it felt like to be a middle school kid and a person in general. You don't want to just sit down and listen. I tried to be as entertaining as possible because I had to entertain myself, too. I also knew what came out of my mouth was of value, so I never spoke when a class was speaking. I didn't have any management issues.

I loved the kids.
They loved me back.
It is a mutual, really good relationship.

Huddles

We would do 'huddles' in class. I would give the students a high rigor task and give them five minutes to work on it independently. After five minutes,

I had students meet me in the hallway if they were done.

If they hadn't finished, I told them not to rush and to keep working. In the beginning, like seven students would come out. My classes were packed. It wasn't legal, I'm sure. Sometimes I had like 42 students because kids said they wanted to be in my class, and I would take them.

If I had a small group outside, we would look at the answer and whisper it together, like "What did you get for number one? How did you get it? When I walked around, a lot of kids have 4. Why do you think they have 4?"

I coached the kids to be master questioners.

Then, I had the small group kids go to someone sitting down and ask them questions. At the end of the class, every kid had one-on-one support and got to talk about the problem with someone else.

I got to a point where I said,
"Hey kids, do you like when it's just me, or when it's one-on-one?"

Sometimes, the entire class would meet outside the classroom. Over time, the kids could unpack never-before-seen problems because they had their own toolboxes.

I remember getting another big visit from the district. This time, I was in full swing. When they left, I thought, "I didn't do anything, but that's how my class runs now." They said, "Wow, your kids have taken ownership of their classroom." I lesson planned very well and very thoughtfully, but the kids read it themselves. They were asking each other questions.

I was just facilitating.
It was amazing.

Huddles became a daily strategy my second year. It shifted the way I created my lesson plan because I had a lot of time. I thought to myself, "What are the things they need? What is a big task that I can create space for them to do what they learned?"

Curriculum: Planning

I was thoughtful, making tweaks, realizing what was going on, getting those drastic test scores, spending the entire summer thinking: "Our kids don't know what value means, that formula means nothing to them, let's have these hands-on projects where they're measuring, let's do all of these things…" My students had done well on quizzes overall. They were learning, but it was not internalized. I had not intentionally spiraled all the power standards.

I just moved from one unit to the next.

I prepared the entire year's sequence and spiraled these standards across classrooms, so the students wouldn't forget what they learned. They saw problems every week, so it wasn't lost to them (i.e., Pythagorean Theorem problems every Monday). Use it or lose it.

I knew how much time we had, and I knew our goal, so I selected which problems we were going to do and how we were going to implement it.

It's all about putting in the work to unpack the curriculum yourself
and recognizing what parts of it you're going to use.

Make sure it's implemented in a way that the students are thinking and considering what's in front of them versus just showing them.

I was still very thoughtful even back in the day about sequencing, but I was not thoughtful about the experiences the students had and upping the rigor of the task in front of the students.

Curriculum: Problems

The curriculum included a lot of practice problems. CAASPP released practice problems, and I love creating math tests. I'm always thinking of things. I thought about the common experiences the students needed across grade levels and what types of daily exposure questions needed to be asked.

We couldn't do everything. Curriculum is important and there is a lot of it. There are more problems than you need. What comes in handy is being very selective as a teacher and asking yourself: "What's the goal of this lesson? What's going to get me there?" We have 45 questions and five example problems, but we can't do all of that. We also have higher rigor word problems.

We did a lot of level tasks. There could be scenarios that weren't very in-depth, but you could ask students: "What questions could be asked from this? What information do you have? What question could be attached to this?" Having the kids doing the thinking and creating these questions as well.

Curriculum: Projects

I spent the summer thinking about the alignment between 6th, 7th and 8th grade,
and certain hands-on projects we need all of our students to do.

I created a hands-on project sequenced for 6th, 7th, and 8th grade, so each grade they experienced a project. The rigor increased each grade, and the standards matched the grade-level standards. The projects were pretty fantastic.

Bedroom Carpet
There was one particular big project. There was a map of a home with different bedrooms and little squares (tiles). The students had to first, on paper, think about the square footage of the carpet needed for two of the rooms. I included the price per square foot at different stores. Some stores had a discount. It was a very layered problem. At the end, the students had to determine exactly how many square feet of carpet they needed, and which store they should purchase it from. I also included a coupon, which made the carpet cheaper, so they had to include their reasoning for their answer. I actually found the stores online, so they could go and shop for it. It was a real thing.

There was a lot of independent time for that project. They would go in their groups and compare their answers. They would try to come to a consensus, but the ultimate consensus was not from me. It was the real-world answer. It was fantastic. At the end, they proved themselves and learned how to cover the space of something.

Swimming Pool
I'd never stressed geometry too much, but I realized they weren't memorizing the formula and they weren't getting the units correct because they didn't know that volume is 3D capacity versus area. It meant nothing to them. They just guessed. Geometry is a real-world thing, so I set out to bring geometry problems to life (i.e., swimming pool cubic inches).

I had a similar layered problem with a swimming pool. The students had to convert the gallons to teaspoons. Then, they measured the water in a measuring cup. If the measurement was correct and the water filled the cup perfectly to the top, you could see the kids' excitement. The project included converting units, measurements, and recognizing value. I also included something like chlorine price, so they chose which is the better deal.

I remember one teacher said: "This is fun, but I'm not doing it in my class. It's going to get too wet." I said: "Oh, yes you are. This isn't for you." She said, "Yes, mother." This stayed in my head because I had to realize you have to respect students that are in different places, and teachers are in different places, too. The way that asked teachers questions changed. I grew a lot from that experience.

I wrote these projects from scratch over the summer. It was a lot of work, and it was for 6th, 7th, and 8th grades. I created each of these with a standard attached. For 6th grade, they didn't have fractional edges, they had squares. There was a triangle in one of them because that's a 6th grade standard. Once we reached the other grade levels, the shapes and figures were irregular and fractional. Project timing depends on sequencing and how the students were doing. For 8th grade, we did it around the same time. 6th and 7th grade might be a little different, but we were all doing it. We had to have flexibility with students and teachers. Ultimately, I wanted to make sure they were not just doing the project at a certain time because everyone else was doing it. It needed to make sense, so it did not always work out that the entire school from 6th to 8th grade was doing a project at the same time.

At the end of the projects, no matter what the students did in collaboration, there had to be proof that they could do it independently. So, there was some form of an independent exit ticket attached. This was different, and it was more time sensitive, but it showed that they could do it on their own.

When we did the projects, we brought them to our department meetings and let teachers know what the students learned from it, what worked, and what the student data looked like.

Managing Time

These projects occurred in one day. I could stretch them out, but I didn't have that kind of time.

We had urgency.

I would say something like: "You have 12 minutes to independently do this. You have another six minutes to get with your group." Time was real. I had to give them time structure. For middle school kids especially, if you tell them to work on something but don't give them the amount of time, they have no compass. Say: "You have two minutes to unpack the problem. Go!" Then, "You have six minutes to come to a consensus. I need to hear a lot of talking. You got it!" Tell the students, "We're doing this whole project in 60 minutes. Can we do it? Yes!" I'm very precise. We're not stretching anything out across weeks. We can come back and revisit.

Saturday School

We had focused Saturday schools where we would invite specific groups of students for particular standards. When I introduced all these pieces, it was very focused and precise. Saturday school was new. We also had the summer bridge to get kids on an accelerated track or get up to speed on material. It was focused and short (about four weeks). Kids could to continue to grow wherever they were. There was one per grade level. I was teaching 8th grade. I would take any student; they didn't have to be in my class. Saturday school focused on one standard. We did a mini test, huddles, and an exit ticket. We had about two and a half hours. We would start off with a little snack, then, we jumped into the work. It wasn't so different than a usual class, except it was very hyper focused on one topic.

Students volunteered to come to Saturday school.

They knew it would help them improve. It was not punitive. They didn't have to come to every Saturday school. Sometimes they didn't need it. I had some students who just wanted to come, and I would have them be assistants. Parents were not resistant to Saturday school at all.

Student Agency

One big piece was student awareness data.

It was up to the students to keep track of their own data.

The goal was, you could stop any kid in the hallway and say, "What was your score on the assessment last year? What's your goal for this year? How many points do you need? Are you getting there?"

How many kids knew this information versus
how many kids didn't know it,
was a piece of data, too.

We had four common grade-level assessments made internally throughout the year that mirrored the CAASPP content. We had the students monitor their progress with a monitoring sheet. In their notebook, each student had a protected sheet with the date of the test, their score, and a space for reflection, including their goals, what they needed to do to get better, and if they were satisfied. They could see which areas they needed to work on. If they struggled in a certain area, they were encouraged to come to Saturday school.

Systemic Changes

I had a fantastic principal. When I first arrived, she was the assistant principal. It was the perfect timing of me being so upset but inspired and excited, and she went from assistant principal to principal.

She was trying to shift every department. The following year, she implemented the monitoring piece and sharing data with the students. It became a school-wide effort, rather than just me trying to do that in the math department. I'm so grateful to her, the timing, the school, and the student population.

She is a former math teacher. She shifted things to data driven. Every department meeting, we had to think: "What are we measuring? When are we measuring it? How are we presenting it?" We were not only presenting it in our small group but twice a year (mid-year and end of year) to the whole school.

The principal would say:
"Take risks.
It doesn't have to be neat.
It doesn't have to be pretty.
We are learning.
We can't do things how we usually do it.
It will work."

The systematic shift encouraged risk taking, student-centered thinking, and data. Maybe some teachers thought she was nuts, but overall, it worked.

You can't do everything all at once, either. We didn't implement the student awareness piece that very first year. We did it the following year because we were still growing. That was a task in itself, having the kids organize and keep up with their scores. Everything happened in a sequence.

Some things needed to be tweaked and honed, but it's actually pretty simple if you think about it. It's not this complicated thing.

The big takeaway is that implementation matters.

I had to focus on that: ask questions and protect thinking time. Once you find what works, stick with it.

Coalition Building

I'd spent the summer working on my personal projects. I'd had about 6-8 months to internally make the shift. When I returned, I had so much work done.

I was the math department head at that point. A few days before school started, I asked my principal if we could meet. I showed her what I had been working on, and she was excited like me.

But the other teachers had only two days before the start of the school year, and not all the teachers felt the same way. They were like, "What? You want us to do what?" "There's no way you're going to get where you need to be in that short amount of time." There were six of us. About half of the teachers were on board. The other half of teachers thought everything was good as it was. Two teachers thought it would be exhausting. There were two teachers who said it was too much. On top of that, those two teachers were in a very impoverished neighborhood with so many things going on.

One of them really took to the approach – I called her my "Mini Me."

There were a lot of materials involved. It could get messy, especially with the water and all the little tiles. I understood that. But I thought about how to truly make sure our students internalize the information. There was no guessing.

There was often blame on the parents or the kids. The students were not equipped to do these types of things, but they just needed practice. I remember the teachers would say the students couldn't get it. The teachers taught it to the students five times over and over again, and they still weren't getting it. That was a red flag.

I said:
"Maybe that approach isn't working if you've done the same thing five times."

If I had a test where half of the kids did well, I'm excited because I had half of them who could teach the others. Even 25% was great.

There was resistance to the huddles and independent thinking, but it wasn't as vocalized. It wasn't until I stepped in the classrooms that I saw it was not happening. There were two things that would happen in these cases:

1) Teachers would come visit my classroom or my "Mini Me's" classroom and watch. She was amazing. She did not play, and she asked urgent questions.
2) Sometimes my Mini Me and/or I would model teach in the other teachers' classes because they said it wouldn't work in their classroom.

To get a glimpse inside of our classrooms, I did something called "Phone in Pocket."

We recorded ourselves on our cell phones
and then shared a 3-minute clip in our meetings.

We started a number of department meetings with this to work on our questioning. You would hear the teacher's questions and ask questions like, "Were there any missed opportunities?"

The big shift in teacher turnaround was because,
ultimately,
teachers want to be successful,
and they want their kids to succeed.

Regardless of if they have a deficit mindset, they genuinely want their kids to do well, and that's sometimes why they teach the same thing five or six times. It's because they're trying. What shifts teachers' mindsets is when they see their students make gains. That's why I always ask, "Where's your exit ticket data? Let's look at it because it's proven."

I remember having one teacher who had the best classroom management ever, but it was too controlled. Those kids could hardly speak. They were too scared to think. The teacher didn't want to let go of the reigns, but when we did it the other way, she said her kids had never scored that high on the exit tickets before—70% of the kids got it. Usually, she had like 20% get it. The teacher saw this work. She felt successful because her kids were successful.

Some teachers said it didn't work in their class, but how many times did they try it? Once? I said: "Let's try it again and get better at it. Don't just throw it out. It's good practice." Sometimes, if it didn't go according to plan, teachers just thought it didn't work versus thinking about what could be wrong with their implementation and what they need to fix for the lesson to transpire.

Team Meetings

I was very convincing.

I was already the math department head before CAASPP, but our meetings weren't as fierce and urgent. I changed our bi-weekly department meetings into real working meetings. We had common small quizzes and common quarter quizzes. We would bring in the data from the assessments and compare it. Then, we would think about it together and say things like: "What are you doing? What do your lessons look like?" We got to the point where we gave each other real, critical feedback, where we had open classroom doors, and we would walk in and see each other's lessons.

Eventually, the team of teachers were co-crafting the assessment questions. We were choosing and tweaking lessons—that's when I got teachers from the other side excited about it because they were part of that assessment creation. They were enthusiastic about the questions.

We were really data driven.
We would look at the data and reteach what we needed to.

The time it took for me to plan the meetings was so intense. I remember always being pressed for times, like five minutes for this topic and 10 minutes for another topic.

Success Indicators

I knew it was working while implementing. I just knew it.

Students stopped saying:
"You haven't taught us this before."

In the past, the students used to say, "We've never done anything like this before" when the only thing I did was switch the variable or make it a vertical versus horizontal table.

I asked another teacher if she saw this happening, and she noticed it, too. Students started using their toolbox and stopped following a step-by-step recipe. They were thinking on their own and unpacked never-before-seen problems and made sense of them.

Another indicator was that the students' level of questions improved.

The openness in the classroom encouraged students to volunteer and share their mistakes and different strategies, unprompted. The goal was to get better and for the whole class community to get better.

We had a student teacher who came for two days. He was very shy. I told him to find his voice. I gave him the lesson, and I watched him teach. My kids weren't used to a teacher explaining everything. He was teaching by himself.

A student raised his hand and said:
"Can I share a mistake I made?"

Another kid's hand went up and asked if he could show the teacher another strategy.

My students refused to be taught and just spoken to.

I was like, "My kids have arrived." I was proud of them. They created space for themselves. And I saw score improvements through the exit tickets and the common assessments.

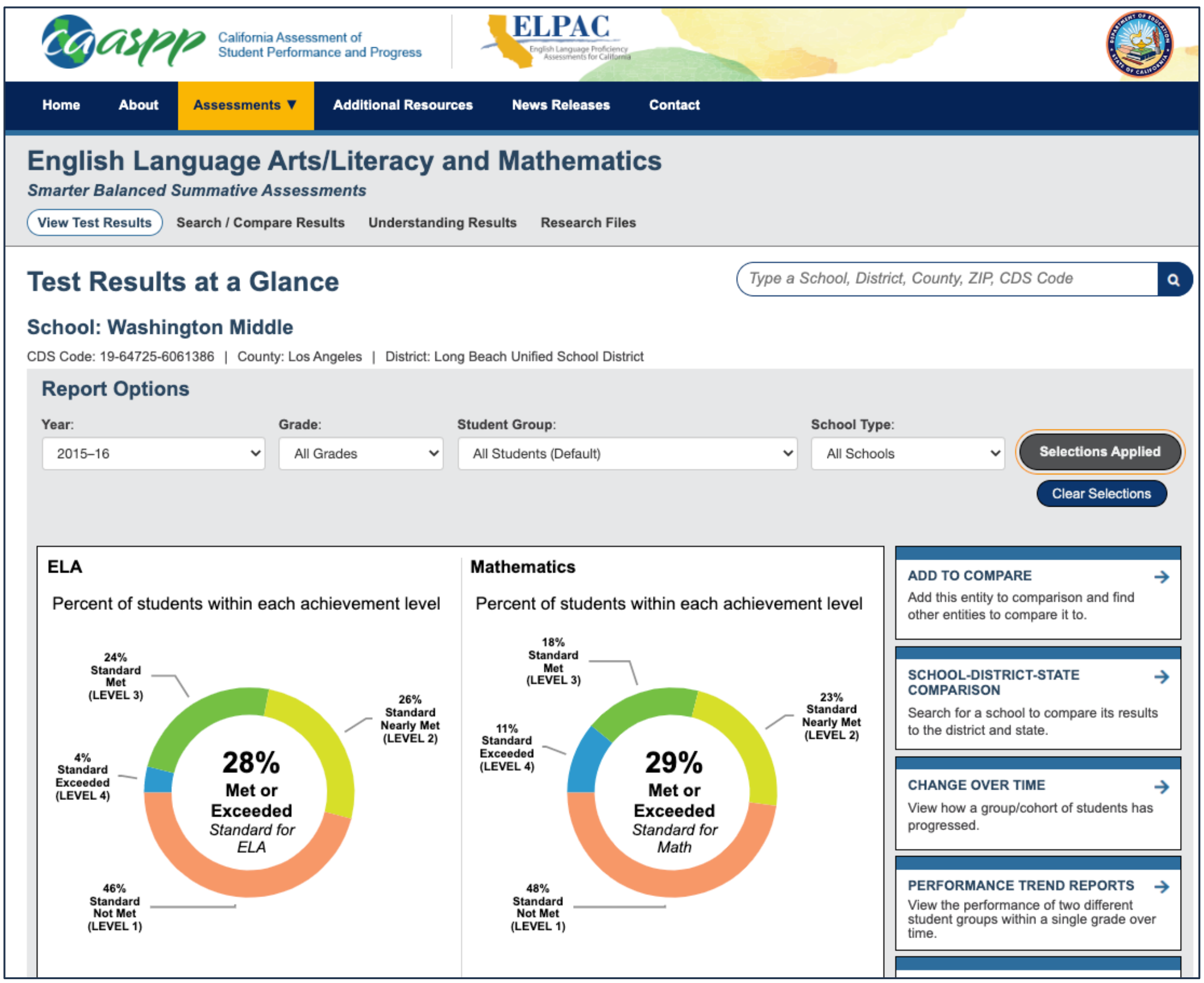
CAASSP, Year 2

At the end of the year, although I was exhausted, I told the principal: "Everything is on the table. If our scores don't increase, I may have to step back. We have done every possible thing we can."

That second year, we had incredible growth. Our scores increased.

> We went from the lowest performing school in the district
> to having the highest increase in scores.

That's when a report came out on our school. They said if we continued like this, we would bypass the other schools.



They did a regression on our school because we were such an outlier. They said we should not be performing like that. We had them come in, and they looked at our department meetings.

They realized the improvements were because of
student data, collaboration, feedback, and practice.

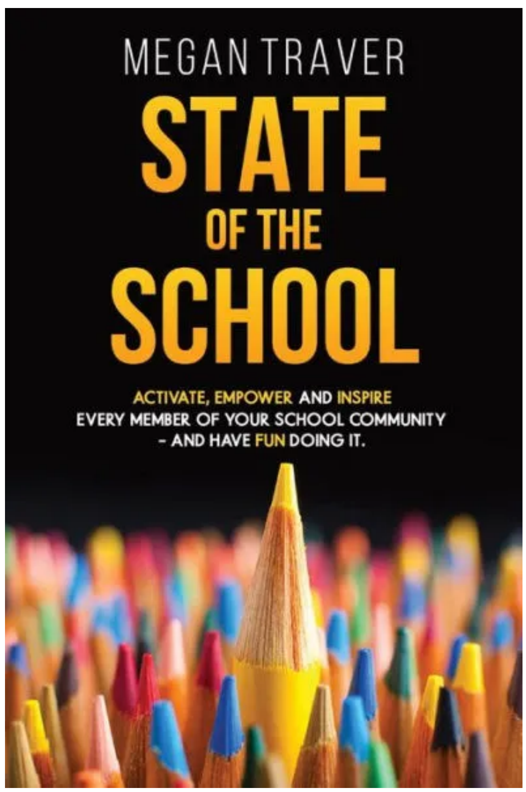
We had folks coming to our school as a lab to see how we run our meetings. My principle and I were asked to share our story with other high-performing principals.

It was the best feeling.
It affirmed that yes, you need a rich curriculum,
but implementation is by far the number one priority.

That's my story of growth.

Postscript

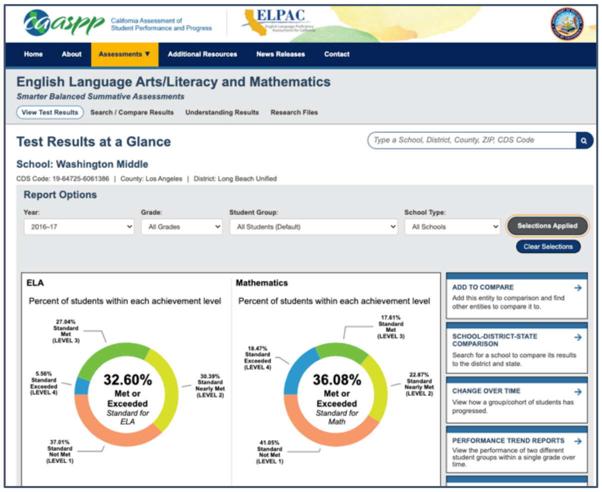
Charity's principal, Megan Traver, went on to author in 2023:

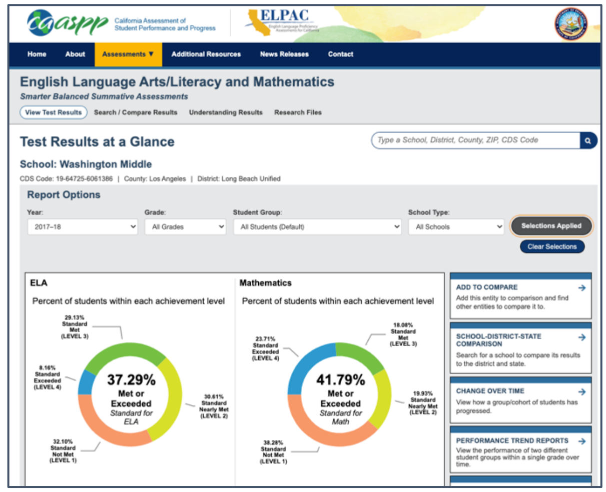


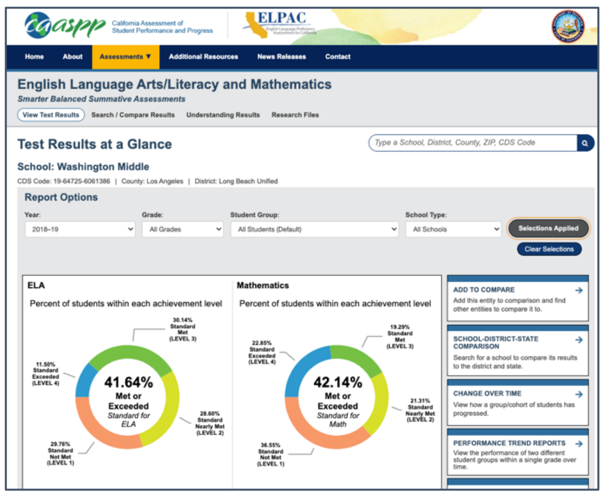
She acknowledged Charity's work:

To Charity Rock – for being the ultimate leadership partner in our work at Washington.
Your fierce belief in our school community, ability to inspire excellence and risk-taking,
and brilliant mind made you an essential catalyst in the change we created.
I am so grateful for you.

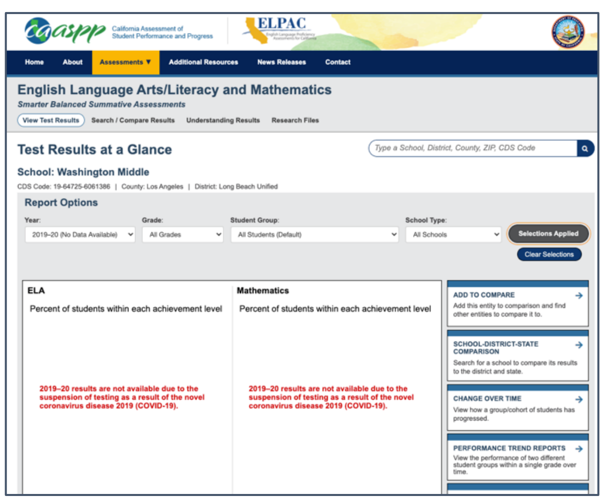
Following their initial success, Washington Middle School mathematics students' scores continued rising in subsequent years.



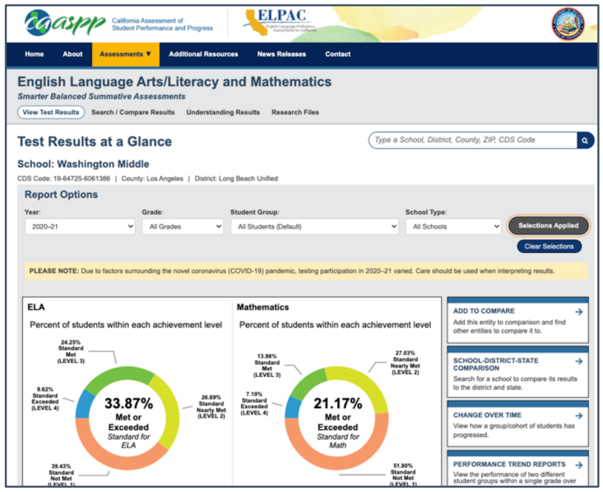




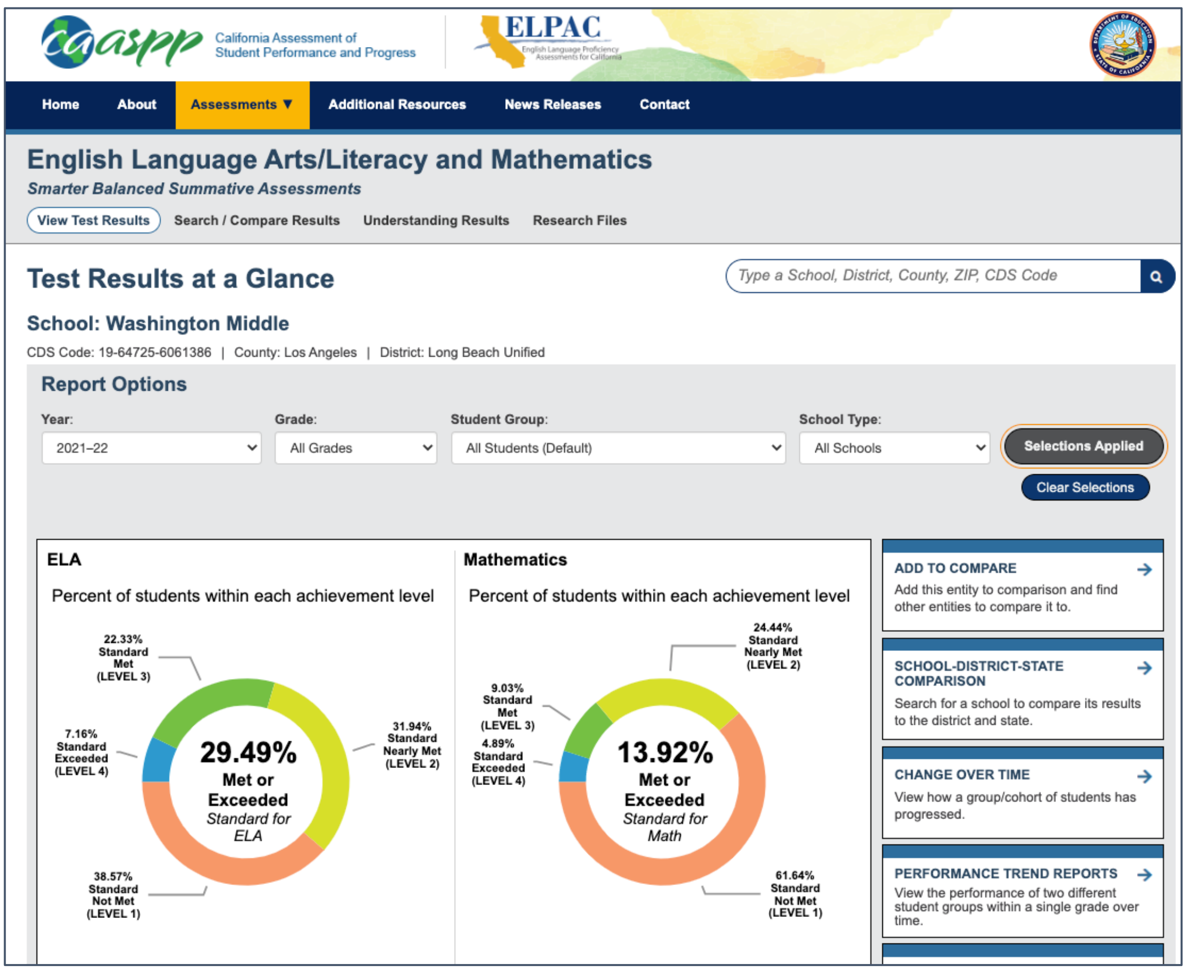
Charity transitioned to the role of [Secondary Math Specialist at Environmental Charter Schools]() in September 2019. CAASPP was suspended during COVID-19...



and results were cautioned the following year.



The most recent CASSP assessment was completed at the end of 2022.



Appendix B: Difficulties and Strategies, by Grade Level

To elaborate on the perceived difficulty ratings by proficiency, this appendix provides commentary from our participants by proficiency level (AB=Advanced Beginner, C=Competent, P=Proficient, E=Expert), and some of the difficulties they encounter, and strategies they use to encourage learning, in subdomains.

Grade 5 Difficulties and Strategies

Operations and Algebraic Thinking

The difficulty ratings suggest that 5th grade Operations and Algebraic Thinking remains an average difficulty domain throughout one's career. Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Operations with Multi-Digit Whole Numbers	Multiplying multi-digit numbers can take 2-3 weeks to master.	When doing multi-digit division, start by asking the students, "Can you get me close?" Instead of using a traditional method of knowing all the multiples of 27, try asking, "What is 27x10?" That method will get you near 400. If a student is trying to get into the 500s and they know 27 x 10, then they can start with 27x10 to do the division. The person next to them might start with 27x20 because they know 27x2 is 54. No student has to start at the same place.
Operations & Algebraic Thinking: Equations	Some students don't understand the equal sign. Some students don't understand if you do something on one side, you have to do it on the other side. Some students see a problem as a whole part instead of two sides that need to be the same. It's difficult for some students to make the connection between the equation and seeing it on a graph. Seeing it graphically is not as simple for some students.	Simplifying the numbers helps 5th graders. Have the students visualize the equation as a seesaw. Draw a line on top of the equal side to help the students see different sides. Erase a number from the equation and ask the students, "How are we going to figure this out?" Some students think any number can fit. Have the students write the equation with the mystery number in a different spot. Ask, "Would another number fit here? Why or why not?" Help the students realize only one number works. Show students how to solve an equation 2-3 different ways. Then, give students the freedom to choose how they want to solve the problem.

		Have students do things on the opposite side of the equal sign. Start graphing the equation by using 2 variables. Use the y intercept and slope.
Word Problems	When students face a problem with numbers and they know the procedure, they can typically solve it. When words are added to the problem, some students can't conceptualize it. They have no clue how to solve it. Some students can't prove the word problems with a model. Students are understanding step 1, step 2, and step 3, but they're not able to apply it to other problems.	Don't immediately tell students exactly how to solve the word problem. Give them a couple of minutes to think about it themselves. Tell students to draw diagrams, highlight keywords and draw pictures to get a better understanding of the word problem. If students are stuck, give them a prompting question to get the ball rolling. Walk around the room and listen to the students' conversations about how they solved the word problem.

Number and Operations in Base Ten

The difficulty ratings suggest that 5th Number and Operations in Base Ten becomes slightly more difficult with experience. Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
The Place Value System	Moving from base 10 to base 60 was difficult. Some 5th grade students haven't mastered whole numbers from 4th grade. Some students don't understand place values.	
Decimals: Addition	It's hard to figure out why the students aren't performing well with adding decimals.	
Integers	Integers are introduced in 5th grade and go until high school. It's important for students to understand the basics, or they will struggle.	
Subtraction		Look at subtraction as distance on a number line so students count up instead of subtracting back.
Subtraction: Borrowing	Some students enter 5th not understanding: why they would borrow how to borrow.	Circulate the classroom to see if students are organizing the problem into sections and/or have the formula written.

	Borrowing doesn't come to students' minds, or students don't know what that number represents.	
Multiplication & Division	Students are supposed to master division in 4th grade, and if not, they struggle. Some students don't know what multiplication is. Most students don't divide to subtract a number. They are basically saying, "I'm guessing and checking." Division is tricky to talk about because students do it in so many different ways.	Instead of doing rote drills with multiplication, incorporate story problems to put everything in context (e.g., you have your best friend [multiplication] and a 'sort-of friend' [addition] over for dinner. You have to send one home. Who are you sending home? The students typically send their 'sort-of friend' home. It's a way to get them to learn the rule of using multiplication first). When doing decimal division or decimal multiplication, include a lesson using estimation instead of just remembering where to put the decimal. Spend 2- or 3-days practicing.
Multiplication & Division: Chunking		First, set up the problem. Before doing any math, ask the students which digit they put first to see where they put the placeholder.

Number and Operations – Fractions

The difficulty ratings suggest that 5th grade Number and Operations – Fractions, remains a difficult domain throughout one's career. Participants offered a number of reasons for their difficulty ratings.

C	They should have learned how to add and subtract fractions with like denominators in 4th grade. There is a tiny bit of review on that but we basically jump right into uncommon denominators, lowest common denominator, least common multiple. There is a lot in terms of vocabulary, greatest common multiple, etc. The process of doing that can be difficult. You need to make sure they have a strong understanding of multiplication. Moving into conceptualization of numbers that they haven't previously looked at. Kids have a difficult time understanding that you can have a part of a whole, but the part has to come from equal parts of a whole. I think that's why we start teaching fractions with a pizza pie. It's a concept of equality too. If you give half of a pie to somebody and give 1/4 of a pie to somebody else, one person got more pie than the other. You have to understand why that is. Comparing fractions, like which fraction is bigger vs smaller, is challenging. As the denominator increases, what happens to the size of the pieces? As the numerator increases or decreases, what happens to the size of the fraction? That's a difficult concept for kids to wrap their heads around. If teaching visually, or using a tape diagram, it's difficult to find the product of two fractions. You are increasing something but it's actually getting smaller.
P	Where the most time is spent. Being very careful to make sure everyone understands concepts. It's rational numbers. We are talking about less than 1. There is an infinite number that lives between 0 and 1. If they can get a good foundation of fractions, the whole statistics piece becomes much easier.

	It encompasses a lot and there are a lot of rules. If students don't know how to simplify or covert improper fractions or mixed numbers, there is a lot that goes into it.
E	Kids understanding fractions becomes very challenging. It depends on how it's taught. Understanding if you are dividing by 1/2, you're actually multiplying by 2. It's a tough concept to understand. Have teachers map things out. Show what it means to divide by 2. You're dividing something into 2 different parts. Visually seeing that helps students understand 3/4 divided by 2 is the same as 3/4 x 1/2. It' a lot of understanding what whole means.

Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Fractions	Fractions are the toughest part of 5th grade math. Most students see fractions for the first time in 5^{th} grade. Fractions are different from the numbers students have been taught. Some students can't identify which fraction is bigger (e.g., ⅛ vs. ¼). Some students struggle with fractions like $^{5}/_{9}$ and $^{3}/_{7}$ because they can't visualize it. If students don't have a clear understanding from the start, it gets tricky. Some students don't know how to give the lowest common form (e.g., ⅛ vs. $^{1}/_{16}$). Some students don't understand the conversion from word problems to fractional ways of thinking. Some students struggle with number lines and fraction bars. Some students haven't learned how to separate the numerator and denominator.	Make sure the students can identify the numerator and denominator. Listen for the students' ability to understand there are equal parts of a whole (e.g., ½ is equal to $^{4}/_{8}$). Relate fractions to sharing (e.g., chocolate bars and pizza). Give students practice finding the common denominator - goes back to multiplication. The concept of coins has helped students understand fractions better. Use number lines. They help students see visually and spatially.

	When comparing fractions, some students don't understand that the wholes must be the same. Putting fractions on number lines tends to be very hard for some students. Some students struggle with improper fractions because its more than a whole number. Some students struggle to finish assignments within the amount of allotted time, or they don't know which questions they need to ask.	
Fractions: Addition & Subtraction	Some students don't have a mental model or visual representation of the numbers they are working with. Some students procedurally solve problems without a deeper understanding. Some teachers teach students to use "cheap tricks" (e.g., The "Butterfly method" to add, subtract, and compare fractions. The trick is OK, but the students don't understand why it works).	Have the students draw the representation (e.g., draw ½ of a pizza, then add another ½). If students understand fractions to begin with, they can usually add and subtract. Start the unit reviewing fractions and mixed numbers. Connect the concept to an experience, but keep examples more abstract, so it's not just the students' own experience. Ex: "I need 2 hours to eat dinner and an hour ½ to take a bath. Then, I read for 30 mins. How much time do I need to get ready for bed?"
Fractions: Multiplication & Division		Make small mistakes while showing a problem to see if students catch where you mess up. Don't discourage any particular strategy. If a student keeps making mistakes, review adding and subtracting fractions. Provide a template, so the students have a procedure to refer to. When you give students a word problem that involves finding an equivalent fraction, specify that they need to find the equivalent fraction, and give them the scaffolding for that. There are a lot of different ways to solve a problem. Ex: ⅞ multiplied by ⅓ - some students can look for the word 'of' in the word problem, while visual learners can draw slices and shade them in.

Measurement and Data

The difficulty ratings suggest that 5th grade Measurement and Data, generally becomes easier with experience. Participants also offered difficulties and strategies for the domains.

Domain	Difficulties	Strategies
Measurement & Data		For a project, have the students find the square footage of a house. Then, have the students find the cheapest carpet for their house by comparing prices at different stores. Include stores with discounts. For a project, have the students convert gallons of a swimming pool to teaspoons. The project can include converting units, measurements, and recognizing value.

Geometry

The difficulty ratings suggest that 5th grade Geometry is more difficult for teachers with less experience. Participants offered a number of reasons for their difficulty ratings, the most important being that less experienced teachers have more difficulty covering the material because they run out of time at the end of the year.

C	It's very conceptual. You have to know a lot of rules and names of shapes. It's very vocabulary heavy: diameter, radius, etc. Words kids need to understand to apply. We historically don't tend to have great results in geometry. Moving more abstract and you don't always have a concrete opportunity. Can be difficult for certain students. A lot of reliance on visualization and not all students find success there.
P	They know how to do coordinate planes really well. Classifying shapes is hard. Understanding rectangles aren't 4 even sides but 4 right angles.

Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Multiplication & Division: Area Model	Some students have never seen an area model before, or they don't know what it means. Students don't understand the concept behind the procedures. If they don't understand the conceptual part, they're not going to be able to tie in previous or future math skills. Many teachers teach procedurally because it's a quick way to solve the problem, and students can master quicker. This is a disservice for future math teachers and for the students because some students don't understand what's happening.	Ask the students to solve a 2 by 2 multiplication problem. If they can successfully complete it, it's not a big leap for them to do a 3 by 3. The ideas build upon each other. Monitor and check in with the students that don't get the 2 by 2 multiplication problem correct. For the students who don't get it, pair them up with a strong peer to learn from. Have the students re-voice their understanding as much as possible.

Grade 6 Difficulties and Strategies

Ratios and Proportional Relationships

The difficulty ratings suggest that 6th grade Ratios and Proportional Relationships remains an average difficulty domain throughout one's career. Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Ratios & Proportions	The language and concepts of ratios is confusing to some students. Some students struggle defining the mathematical words in a problem. Some students have a hard time substituting for a variable. With unit rate, some students forget how to label the graph.	Review prerequisite skills of fractions. Ask questions like, "How do we show this on a graph? How do we locate it on a graph? What does this point mean on the graph?" Define what "sum" and "product" mean. Break down the questions with the students. Have students complete a project that crosses mathematical strands, so you can reference those topics throughout the course (e.g., have students go from ratio conversions to parts of a circle. The students don't even know they are doing equivalent fractions, so when they get to equivalent fractions, they have already done them and realize it is easy for them).

The Number System

The difficulty ratings suggest that 6th grade Number System becomes slightly more difficult with experience. Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
The Number System; Negative Numbers	If students struggle with negative numbers, they are going to struggle with future topics.	
Integers	Integers are new for 6th graders, so it can be an abrupt transition for the students.	Get an idea of the students' prior knowledge with integers. Connect integers to the previous grade level if you can't connect them to the previous unit.

Decimals	The intangible part is hard for some students, but most students know $1.25 is more than a dollar.	Most students grasp the concept of money because it's concrete, and it helps them understand if an answer is sensible.
Exponents	Some students don't understand substitution to evaluate an expression. A lot of teaching time is spent talking about common misconceptions (e.g., students will do 3x2 instead of 3 squared.)	

Expressions and Equations

The difficulty ratings suggest that 6th grade Expressions and Equations, becomes significantly more difficult with experience. Participants offered a number of reasons for their difficulty ratings.

C	Kids enjoy balancing equations. They like the concept equality and balance. Whatever I do on the right side of the equation, I have to do on the left. I've experienced a lot of success with kids when it comes to expressions and equations. Moving into abstract. Understanding there are numbers that exist that you can't see at the moment. Understanding variable value.
P	It's so algebraic. We start with manipulatives but it's hard to apply this to a real-life situation for them. A lot of abstract. Students struggle with expressions and equations. Teaching it feels easy in a sense but not really because people are intimidated with how it connects to algebra. Students are unable to solve equations. At the 6th grade level, they aren't necessarily solving but they do start applications. That's when they feel 'Oh my gosh there are letters in my math.' It takes them outside of themselves.
E	It's more abstract.

Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Fractions	Students are always behind on fractions. Some students don't see fractions as division. Some teachers don't want to spend time on another fractions lesson because most students have learned it for 4 years.	It helps when students are allowed to use calculators. If a student has misconceptions, provide more examples and break it down. Draw a picture, use non-examples, or have other students explain it to them.

Fractions: Division	Some students miss steps, like locating a rational number on the number line. Common errors occur when students simplify fractions and work with numerators. Most students were exposed to multiplying fractions in 5th grade, but dividing fractions can be more difficult.	Use fraction tiles, so the students can visualize it. Some teachers use phrases like "keep, change, flip" to help students remember how to divide fractions. Some teachers are against this strategy because the kids are just remembering a phrase and not understanding the *why*.
Expressions & Equations	Some 6th graders are still very literal, so it's hard for them to understand the abstract. There can be more than one solution for problems. Once some students find a way that works for them, they don't want to try other solutions.	

Geometry

The difficulty ratings suggest that 6th grade Geometry becomes significantly easier with experience. Participants offered a number of reasons for their difficulty ratings.

C	They start to see area, volume, etc. as concrete things. Formulas that come into play. Using a formula is simple plug and play for students.
P	Surface area is involved.

Participants also offered difficulties and strategies for the domain.

Domain	Difficulties	Strategies
Geometry	When doing 3D polyhedral, some students need the model, and some students don't. The formulas involved require memorization. Some students don't get units correct because they don't know that volume is 3D capacity versus area. It means nothing to them, so they just guess.	Students see things differently, so it's okay if they need a number line, a block, or a paper shape. Just because some students use different strategies doesn't mean they are below grade level. Bring geometry problems to life (e.g., cubic inches for swimming pools). Include Pythagorean Theorem problems weekly, so students get regular exposure and don't forget how to do it.

Statistics and Probability

The difficulty ratings suggest that 6th grade Statistics and Probability becomes significantly easier with experience. Participants offered a number of reasons for their difficulty ratings.

AB	Students really like statistics. They were good with that.
C	It's a fairly new subject and skill. It's heavy on vocabulary and understanding what the different things mean. Real world centered. A lot of stats and probs questions are based in real world context. That adds another layer of rigor. Kids have to read and extract information and understand how to apply. Our teachers have not spent as much time on stats for whatever reason. Some of our standards shifted to 6th grade in NY. Sometimes it's seen as an isolated domain. Less time is spent on it, it's more real world, our teachers aren't familiar with the vocabulary, etc. If it were taught correctly, it wouldn't be that challenging. There is less arithmetic so it's more conceptual in a lot of ways. Very situational. Moving into the world of word problems and conceptual thinking. Having to understand the scenario.
P	We never finish curriculum and it's always last. I think it's hardest for kids because they aren't exposed.

Grade 7 Difficulties and Strategies

Ratios and Proportional Relationships

The difficulty ratings suggest that 7th grade Ratios and Proportional Relationships, becomes slightly more difficult throughout one's career. Participants offered a number of reasons for their difficulty ratings.

AB	There are three things we normally talk about: Ratio table, Equations, and how to transfer those into a graph. The graph part, kids really struggle with. That's a key thing we normally try to stress. The ratio table is fair enough and easy to understand how to find the unit rate. They don't understand how a constant number makes a ratio proportional. It comes to equations and how to identify y, x, and the constant. They struggle between those 3 equations and sometimes they mix them up. You have to do a continuous practice between equations and the graph part.
C	They've experienced ratios and proportions for about 2 years. Now, it gets more conceptual. They have to apply proportional thinking to real world math problems. Tends to be more challenging. Proportional relationships take up a ton of time. We tend to see kids struggle there. I applied it a lot to real world scenarios. They knew they were working with a lot of fractions. Asking them, "Why does this unit make sense to this story?" That was something that clicked with them.
P	We have manipulatives to help students with that but it's moving towards an abstract way of thinking. In elementary school, there is a right and wrong answer. They still have a mindset of what the answer should look like. Two people can have the same correct ratio, but the proportion looks different. That messes with their head. One of them must be wrong. Big portion of the state exam. It's a lot of fear that 7th graders have given how badly 6th grade ratios went. Every time I taught 7th grade ratios they would say, 'Ratios again?' It's that mindset is negative.

Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Ratios & Proportional Relationships	Some students struggle to keep things in proportion as they stretch different dimensions, and they often get a number that isn't what they expect (e.g., if you shrink a 2D shape, some students want to say it will have half of the area). When the students actually do the math, it's hard for them to see why. Then, if you reduce it by 3, some students don't know how that's going to affect the area or the 3D volume because it's more complex than just finding equivalent ratios. Some students struggle putting ratio tables onto a graph. Some students struggle multiplying and dividing on the ratio tables. One student called the ratio table a different name.	Ratio tables aren't necessarily called out in the curriculum, but some of the strategies are visual and helpful for students. Some students have used the ratio table before. Get a sense if the students have heard of ratio tables at the beginning of the lesson. Find a common ground for students to understand the concept (e.g., the ratio between the boys and girls in the class). Ask students if they've seen graphs and know the quadrants. Then, ask them how to label the y and x axis. Have students put the ratio table on a graph.
Graphing	Some students don't know what a point is. Some students confuse the x and y axis (independent and dependent variables).	The students should know the x and y axis and where the negatives and positives are. Have students draw their own graph and label it. Relate the graph to a number line or temperature (e.g., when the temperature drops, it goes to the negatives.) If x is at 2 and y is at 24, ask students where y would be if x is at 1. The points need to create a straight line for it to be proportional.

The Number System

The difficulty ratings suggest that 7th grade Number System remains an average difficulty domain throughout one's career. Participants offered a number of reasons for their difficulty ratings.

C	We now allow students to use a calculator throughout 7th grade. Whereas, in 6th grade, it's 50 50 on how often they are allowed to use a calculator. It makes the number system domain a little more accessible. Easy because we started the school year off with the number system. Heavy on the number lines, location of negative and positive numbers and being able to elaborate through their findings. Identifying the fractions on a number line was a struggle.
P	This is the first time rational numbers are introduced. Only reason it's not #1 is because they are allowed to use a calculator. It's not as challenging to do it all by hand anymore. Students struggled with the most on the state exam. Every time we say fractions they want to freak out.

Expressions and Equations

The difficulty ratings suggest that 7th grade Expressions and Equations, remains an average difficulty domain throughout one's career. Participants offered a number of reasons for their difficulty ratings.

AB	Kids struggle with understanding. When they start working with one step problems, it's much easier. When we get to two step and three step, there are word problems. Word problems into your own equation and inequalities is where they really struggle. You have to deal with the word problem and kids are not really fluent with problems like that. Sometimes it's also their reading level. If they aren't on a higher reading level, they struggle breaking down each word problem and breaking it down into an equation or expression. Sometimes inequalities because it's in the same topic.
C	Getting them ready for algebra. Having them isolate the variable, moving one constant to the other side, etc. was really engaging for the students.
P	There are word problems involved. Difficult especially when we get into the word problems. If the expression looks different, but you can solve it the same way, kids freak out. Math is creative. We take the creativity out of math as they go through school. This is where creativity comes back into play. Some students struggle with that. Even the high smart kids struggle with that.

Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Expressions & Equations	More than 50% of students don’t understand that a letter is a number. Variables are a new idea for most students. Thus far, students have mostly dealt with one-step equations. Students struggle moving x to the other side of the equation (e.g., 3x+2=26+x.) Students have to show their work, and some students push back. There are different ways to solve for a variable. Some students don’t know what to do when they receive a calculator. Students struggle putting rational numbers into expressions. Students struggle with equations when they don't know which fraction is smaller or bigger. Some students don’t understand distributing a number into an expression.	Look at the steps the students are using in the equation, especially the students’ first step. The first step is key. Have the students find the word “equal” in the question and tell you what that means. Solve problems in different ways (e.g., 3x+6=24: you can do the distributive property or divide on both sides). Show students both routes and let them decide which works for them. Pay attention to both options when looking at the students' work. Use of visuals and fraction tiles because many students need to see to understand. Use real world examples and incorporate hands-on activities. Money can help students understand rational numbers and parts of a whole (e.g., $1 is 100 cents. ¼ is .25.)
Variables in Word Problems	Some students struggle figuring out the variable in a word problem. Some students don’t want to read, or their reading level is low. Some students go straight to equation writing instead of underlining key words and going over the entire word problem.	Work on the steps of solving variable problems before putting them into a word problem. Students should read the word problem at least twice and underline key words, create a key from the words, and use the key to help solve the problem. Look at the key words that the students underline in the word problem. Get students to understand that the variable is never a number that is given. Stress what the question is asking – that is where the variable lies. Ask the students to find the variable within the word problem.

	When some students aren't getting it, they write an expression instead of an equation. They will get parts of the problem but not structure it correctly.	Ask the students to create their own equation. Ask the students which side of the equal sign the total should be. Ask the students if they are subtracting or adding in the problem.
Quadratic Integers	Some students don't understand what a square root is.	

Geometry

The difficulty ratings suggest that 7th grade Geometry remains an average difficulty domain throughout one's career. Participants offered a number of reasons for their difficulty ratings.

C	Not a ton of geometry in 7th grade. The standards are pretty simple. We haven't touched on it that much and when we did, my students were having a hard time remembering the name of a 5-sided shape. Struggling with finding the area and applying units. Having a hard time decomposing the shapes.
P	I haven't done a lot of constructions with them and it's very difficult. I'm not sure how it's assessed on MCAP. They have a tough time seeing those relationships. It's heavy vocab. They struggle a lot with complementary, supplementary, vertical, etc. It's a constant, "What do these two equal?"

Statistics and Probability

The difficulty ratings suggest that 7th grade Statistics and Probability, remains an average difficulty domain throughout one's career. Participants offered a number of reasons for their difficulty ratings.

C	Same reasons as 6th grade. We didn't touch on it that much. They had a hard time working with box and wisker plots. They didn't struggle with finding the mean, average, median, etc. When it came to the conceptual understanding of things, they were having a hard time explaining. Understanding populations. A lot more technical vocab. Understanding how the vocab relates to the word problem can be difficult. They have to understand the words that are being used to understand the problem to solve.

Grade 8 Difficulties and Strategies

The Number System

The difficulty ratings suggest that 8th grade Number System, remains one of the easiest domains throughout one's career. Participants offered a number of reasons for their difficulty ratings.

C	We don't spend much time on rational and irrational numbers. Could be 2 or 3 lessons in the whole year. Easiest because it's pretty much memorization. Rounding was weirdly low in our FSA.
P	The number sense. Without a calculator. The irrational square roots in-between 1 and 2 whole numbers, they struggle.
E	More procedural."

Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Integers	8th grade switches to more procedural and conceptual learning.	Number lines help some students understand integers conceptually. Give students an activity with a hot air balloon. If you add a balloon, you go up the line. If you add sandbags, you go down the line.
Negative Numbers	A lot of students are deficient in negative numbers.	Desmos calculators can help students with negative number problems.

Expressions and Equations

The difficulty ratings suggest that 8th grade Expressions and Equations becomes slightly more difficult with experience. Participants also offered difficulties and strategies for subdomains.

Subdomain	Difficulties	Strategies
Expressions & Equations	Some students really struggle with exponents.	Tell students variables are just a number. Make a connection with the real world (e.g., The Metro: you have $40 on your metro card and each time you use it, it's $2.50). Then, spiral back to previous knowledge and make the connection to proportional relationships. There are different ways to think about expressions with elevation (e.g., if you go down in an elevator, that's negative. The basement is underground, which is the base. On an airplane, you go above sea level, so that's positive).
Algebra	The class is challenging. Algebra is difficult to pace. Some students don't build as quickly. Students need to solve problems a certain way because that's what high schools expect them to do.	
Commutative Property	Commutative property is fundamental to a lot of 8th grade math. Commutative property is a struggle that comes from gaps of remote learning. If students don't have a basic understanding of the commutative property, it can be a 4-week process. There is so much information to cover. Students should've learned the commutative property in 7th grade. The commutative property becomes more present as students progress in math.	Use sprinkling opportunities to make up for learning gaps and refresh topics.

	Many teachers constantly reteach the commutative property. Teachers must decide if it's worth it to spend 3-5 days reteaching.	
Quadratics	Some students don't know how to approach a question they haven't seen before (e.g., solving quadratics by factoring.) Some students don't understand simplifying radicals.	Put quadratics in context of the real world (e.g., think about what happens when you throw a ball. Talk about the shape of a curve. The ball doesn't go in a straight line, it makes an arc. Establish what the arc is. How many times does the ball hit the ground if you kick it)? Once students understand the concept, introduce the vocabulary. The curve is the quadratic. Have students think about the characteristics of a quadratic and ask them to come up with other examples themselves. Ask students to give the factors of quadratics. It gets to the same concept. Look at parabolas before looking at equations of parabolas.

Functions

The difficulty ratings suggest that 8th grade Ratios and Proportional Relationships, becomes significantly more difficult throughout one's career. Participants offered a number of reasons for their difficulty ratings.

C	Easy for teachers to teach. It's like a vending machine. You put something in and take something out. It's a really cool visual that is easy for kids to see. You substitute in a number and another number pops out. Students eventually get good at identifying what a function is and substituting a number to find something else. They are trying to evaluate functions. They model the relationships between the different quantities, the difference in the model and the actual numbers. Sometimes they will understand the model and how the model relates to a situation, but they don't necessarily understand how the specific quantities or how you evaluate based on the model.
P	It's more abstract. It's not hard. If we have really strong bedrock foundations, it's not hard. But if we have some sand, it becomes increasingly challenging to get students to master that concept. We use functions all the time. You can't get a cellphone plan without using functions. I guarantee no adult sees it as a function and sets it out to solve a function. You are doing algebra 1 but we don't think about it that way. A teacher was planning a wedding. Every warm up question all year was a math question about her wedding. She was throwing in seating charts, etc. It was an application they finally started to buy into. The real-world situations help. First time they are seeing it. Most abstract. What is a function and what is not a function doesn't have a lot to ground them from prior knowledge. They can't connect it to the real world.
E	None of these topics are difficult if taught correctly.

Geometry

The difficulty ratings suggest that 8th grade Geometry becomes significantly easier throughout one's career. Participants offered a number of reasons for their difficulty ratings.

The chart illustrates how the experience of teachers positively correlates with the ease of teaching geometry. Teachers tend to allocate more time to algebra instruction, potentially limiting the depth of geometry coverage. Students struggle with knowing which formulas are applicable in certain problems and where to apply them. Additionally, the introduction of new and abstract concepts can prove challenging, as students adjust to the visual thinking required by geometry.

C	Has been a challenge due to the fact we are spending more time on algebra. Kids get geometry or they don't. You'll have kids that do horrible in algebra and then crush geometry. It's pretty straight forward. It's easy to teach. Didn't touch on them for algebra. Have to understand what formulas to use and where.
P	There are a lot of new geometry concepts here. It's a lot in 8th grade. Pythagorean theorem ties into solving equations. If you can't solve equations, you can't do that.
E	Easy for them to understand even though formulas are still prevalent. The formulas involved the students have to memorize. They shouldn't have to memorize a formula.

Statistics and Probability

The difficulty ratings suggest that 8th grade Geometry becomes significantly easier throughout one's career.